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THE PSYCHOLOGY OF THE COMMON BRANCHES

BY

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EDITOR'S INTRODUCTION

The present volume is a very interesting as well as a very successful attempt to apply the knowledge which we have recently accumulated in the scientific applications of psychology to the concrete problems of instruction in the elementary school. It is neither a scientific treatise on psychology nor a book of special methods, though embracing something of the content of each. Instead, the present volume occupies a field lying between the two, being a presentation of the psychological principles underlying the most effective instruction in the commonly recognized subjects of the elementary-school curriculum.

Textbooks on general and even applied psychology almost of necessity deal with the fundamental principles and generalizations of the science, and the applications are usually so remote from the practical problems of the teacher that the application to classroom procedure is seldom made. On the other hand, our books on general and special methods, while often very helpful and suggestive in their way, tend nevertheless to confine their attention to schoolroom devices and certain general pedagogical principles, and do not serve to develop in the teacher any tendency to seek out or formulate the reasons for the special methods which are being followed.

In between these two extremes of psychology on the one hand and special or general methods on the other lie two new fields in applied psychology — genetic psychology, which attempts to organize psychological knowledge in terms of mental evolution, and the psychology of the process of learning to write, read, spell, calculate, etc. Genetic psychology lies nearer to the pure psychology end, and the psychology of learning lies nearer to the methods end.

The present volume is a treatise on the second of these two intermediate fields, namely, the psychology of the learning process, and as applied to the so-called fundamental subjects of the elementary-school course. For three of the subjects applications are made to the high-school field as well. The attempt has been made so to present the fundamental psychological facts as to give a clearer understanding of the pupils' difficulties, the best methods for overcoming these, and the psychological reasons for the special methods to be pursued. A leading purpose has been to show the teacher how to follow or apply methods intelligently, and especially how to adapt methods to the peculiar needs of the individual pupil.

The order of treatment deserves mention, as it follows the well-known pedagogical principle of proceeding from the simple to the complex. The volume begins with the psychology involved in the teaching of handwriting, which requires a relatively simple form of learning of the sensori-motor type. This is

followed by the psychology involved in teaching drawing, reading, and music, which are good examples of what is known as "perceptual" learning. Next follows the psychology of instruction in spelling, which is an example of the fixing of associations. This in turn is followed by history and geography, which are examples of a more difficult teaching process, as they involve the organization and extension of experience through the use of the imagination. Next comes the psychology of teaching mathematics, with applications to instruction in algebra and geometry as well as arithmetic, studies which involve abstract thinking. The volume closes with a consideration of the psychological principles involved in giving instruction in the natural sciences, which involve the ability to generalize upon the basis of accumulated experience. Taken in this order and together, the different elementary-school subjects offer an excellent basis for the presentation of the chief types of the learning process.

Such a volume as this would not have been possible a decade ago, and the fact that it is possible to-day serves to illustrate the rapid advances being made in experimental education. It has required much technical knowledge and considerable skill in presentation to organize, in a simple and non-technical form, the practical results of the many scientific studies of the learning process which have so far been made. As a textbook in courses in elementary education in normal schools and colleges this volume should find large

usefulness, and it should also prove of much service as a simple treatise on the psychology of elementary-school methods for teachers organized into reading circles or study clubs.

ELLWOOD P. CUBBERLEY.

PREFACE

DURING recent years there has been a good deal of investigation for the purpose of analyzing the learning processes which are characteristic of the common branches. The publication of E. B. Huey's *Psychology and Pedagogy of Reading* marked the first attempt to bring together some of this material and present it in such a form that it would be serviceable to teachers. Additional material on reading has become available, and considerable work has also been done on writing and spelling, and a little on number. The accumulation of data has reached a point which seems to justify a comprehensive account of the psychology of the common branches. The amount of experimental foundation for the analysis of the different subjects varies from a fairly satisfactory amount down to nothing. In the case of the subjects which have not been directly investigated, however, there has been experimentation upon the mental processes which are very obviously involved in them — as in the case of history — so that most of the discussion has some experimental foundation, direct or indirect. No apology is made for the fact that a part of the analysis is in the nature of inference from general principles which have been thus determined, beyond the acknowledgment that such analysis is subject to later verification or revision by

direct experimentation. One benefit to be derived from making a formulation, when experimentation is not yet complete, is that this serves to define some of the problems and thus to stimulate investigation.

I take pleasure in using this opportunity to make certain general and specific acknowledgments. For the early direction and the continued encouragement in the pursuit of psychology I am indebted to my college teachers, Professors A. C. Armstrong and Raymond Dodge. Both in my work as a graduate student and as an instructor I have been closely associated with Professor Charles H. Judd, and owe the special direction of my efforts, and much of the formation of my views, to his influence. He has also given helpful criticism upon the manuscript of this book. Professor S. C. Parker has given me valuable criticism and stimulus, and I am indebted to Professor W. L. Miller for carefully reading the whole manuscript. To many others I am indebted, either for their general influence and stimulus or for suggestions concerning the manuscript. The help of all these, whether mentioned by name or not, I gratefully acknowledge.

F. N. F.

CHICAGO, *March 1, 1916.*

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THE PSYCHOLOGY OF THE COMMON BRANCHES

CHAPTER I

INTRODUCTION

WHEN the child takes pencil and paper and makes his first attempt to write, he is beginning a long course of training in the particular form of learning which we call "sensori-motor." His first attempt is to make a mark which shall look like the marks which are set before him as a copy, and the progress which he makes as he practices consists in getting better and better control over his movements so as to be able to produce letters and words which look more and more like the copy. Writing does not require in any large degree the exercise of memory, of imagination, or of reasoning. It requires, as the term "sensori-motor" implies, the adaptation of movements to an object which is perceived. In learning to read, on the other hand, movements play a minor part compared with the acquirement of the ability to recognize quickly and correctly the combinations of letters as they appear in words, and to give them their true meaning. The understanding of number requires that one shall disregard most of the qualities of objects and pay atten-

tion only to their number. Accordingly, we say that learning number involves the process of abstraction. Similarly, other subjects of the curriculum furnish illustrations of other forms of learning.

There is a great deal of variation in the efficiency with which these various kinds of learning are carried on in the school. Some children learn rapidly, others slowly. Some waste effort, others economize it. Some reach a high degree of proficiency, others remain permanently at a low level. If we compare, not individual children, but different classes or schools or school systems doing the same work, large differences in efficiency still appear.

A certain proportion of the differences between individual children, and even between different classes or schools, may be attributed to differences in native ability, but in large measure a child learns economically and well, or the reverse, according as his learning is supervised well or poorly. The child may stumble along and by blind trial learn how to improve. In some forms of learning, in fact, random trial forms a fairly large part of the method. But when the child's efforts are not intelligently guided, the amount of blind trial is larger than necessary, and much time is wasted in consequence. Experiments in supervised study have shown that, even in the high school, the pupil needs considerable help from the teacher to show him how to go about his work in its more general aspects, and to meet the special difficulties of each subject. How much

more is the elementary-school pupil in need of skilled supervision!

In order to supervise the child's learning most effectively, it is necessary to know what mental changes take place in the different forms of learning which he undertakes. Merely to know the result which is aimed at is not sufficient — important as this is. One can learn in a short time to distinguish between good and poor writing, between an adequate and an inadequate understanding of history or geography, or between different degrees of proficiency in the pursuit of a science. A clerk can correct examination papers in arithmetic or spelling. These forms of discrimination make up the art of the examiner. When carried to a high degree of perfection in the development of standard tests of attainment, this art becomes more difficult, and it becomes possible to use the results in directing the pupil's efforts. But in the main the art of the teacher is the far more delicate one of following the changes which take place in the mind of the pupil, and of bringing to bear his influence upon them to produce the highest degree of improvement with the least expenditure of effort. In order to understand the process which is going on in the child's mind when he learns the various branches, it is necessary to have a conception of the typical learning processes which are involved in these branches.

It might be supposed that, having gone through the learning processes himself as a child, the teacher would

have sufficient insight into their nature from his memory of his own efforts, without making any further study of them. This memory of one's early struggles, however, is not sufficient. In the first place, memory has become very dim with time. From great familiarity with the tasks which the child has to meet, one becomes subject to the illusion that they are easy, and that the only requisite for their performance is good-will. Let the teacher try one or two of the tasks which are suggested in the following pages as illustrations of learning of a sensori-motor or a perceptual sort, — such as writing with the left hand or the toes, or gaining a correct apprehension of the figure shown in chapter III, — and he will get something of the baffled, helpless feeling which possesses the child which will set him to searching for some way to help the child to find out *how* to meet the demands which are laid upon him.

A second reason why the teacher's early experience is not enough to fit him for the task of guiding the child in his learning is that the learning attitude and the teaching attitude are different. The learner is absorbed mainly in the result which he wishes to accomplish, and he does not make much analysis of the means by which he reaches his goal. In fact, he sometimes will reach his goal more quickly if he does not analyze the process very much. But the teacher must have the analytical attitude. He must be continually thinking about how the act which he is teaching can best be performed. For example, the child in

writing does not pay attention to the movement so as to know what simple movements compose it and how they work together, but this information helps the teacher to give the child exercises which will emphasize the proper components.

While the analysis of the learning process in the common branches, which occupies the following chapters, does not go into the details of the methods of teaching, the constant aim is to give practical assistance to the teacher by giving the basis for an intelligent use of methods. The different school subjects taken together furnish a pretty comprehensive set of examples of the various kinds of learning of which the human mind is capable. The subjects are so arranged that the simpler, more elementary, and less intellectual forms of learning come first, and the higher types in their order. The topics taken together furnish a fairly complete account of psychology in its active or functional aspect, apart from the instincts and emotions.

CHAPTER II

HANDWRITING: LEARNING CHIEFLY OF A SENSORI-MOTOR CHARACTER

The sensori-motor form of learning of which handwriting is an example has for its purpose the development of a motor habit.[1] A motor habit exists when such a connection has been formed between a set of stimuli[2] and the movements which are made in response to them, that the movements follow readily upon the stimulus without the necessity of being guided and directed in detail. Thus, when a person wishes to write a word, it is not necessary to think about the movements of the fingers or the arm which produce the letters, nor is it necessary to think of the details of the letters themselves. This is true of all sorts of sensori-motor habits.

The composition and development of the writing habit

The problems of teaching writing are concerned very largely with the development of an efficient movement. We may, then, turn first to a consideration of the com-

[1] A sensori-motor activity is one which consists in the adaptation of a movement to a stimulus.

[2] A stimulus is any object or event in the world outside the individual which is capable of causing a sensation, an idea, or a movement — or, in other words, of producing a response.

position of the writing movement and of the way in which it is developed.

The writing movement is a very complex one. It is composed of a great many elementary or simple movements. In the first place, there is the movement of the fingers and of the hand. Even if the letters are not formed by finger movements, the fingers must be employed in holding the pen. In doing this the first and second fingers are separated in their action from the third and fourth. The first two fingers with the thumb hold the pen, while the third and fourth fingers support the weight of the hand. This division among the fingers is something which is more or less artificial; that is, it is not the natural manner in which the child grasps objects. The most primitive method of grasping an object, which is seen not merely in the child but also in the monkey, is to fold the four fingers about it without using the thumb. The ability to place the thumb against the tips of the fingers in grasping an object is a distinctly human one and one which develops later in the child than the other form. The mere act of holding the pen in the conventional manner, then, is a matter of considerable difficulty to the child.

In addition to the adjustments or the movements of the fingers there are movements of the various arm joints. The wrist is frequently moved from side to side during the writing of the letters of a word. This is not a movement which is commonly approved by writing-

masters, but it is very generally used. The hand also is carried along the line by the movement of the elbow and of the shoulder, together with the wrist. Whether the greater part of the movement is made at the elbow joint or at the shoulder will depend on the height to which the arm is raised. If the arm hangs at the side, none of the movement is made at the elbow. The movement by which the letters are formed may also be made partly at the shoulder joint. This movement is sometimes called the forearm movement, but it is not, as a matter of fact, made by the forearm. It may be made with the arm resting on the muscle pad of the forearm, but the muscles which produce the movement are at the shoulder. The muscles in the forearm, on the contrary, control the movements of the fingers, as may be readily verified by placing the other hand on the forearm and then closing the fingers.

These component movements coöperate in the total movement. The writing movement to be most efficient should be made up of a combination of these various component movements, since each of them is particularly suited to carry on some part of the whole task. The side-to-side movement of the forearm, revolving about the muscle pad as a center, should carry the hand along from the left to the right side of the paper; and this movement should go on while the letters are being formed so that the writing does not have to be interrupted. The movement of the whole arm from the shoulder resting on the muscle pad of the forearm

is well suited to carry the hand up and down so as to play the chief part in making the upward and downward strokes. It is probable that this large movement produces a rhythm and regularity in the writing and a smoothness of stroke which gives the written page a good appearance.

Furthermore, it is probable that if a considerable part of the movement is made with the arm, it is not so fatiguing as when the fingers do the whole work. The finer parts of the letters can most easily be made with the fingers. Although the attempt is commonly made entirely to exclude finger movement, it rarely succeeds, and the most rational course is to recognize that the fingers, as well as the arm, have their own part to play.

Overslant at the end of the line may be corrected by pronation. Besides these components of the writing movement which produce the words, there is one which is sometimes used to correct a fault resulting from the mechanical conditions of writing. As the hand is moved forward by the rotation of the forearm about the muscle pad or the elbow as a center, the pen points more and more to the right. This is illustrated in Figure 1. This causes, unless it is corrected, an overslant of the writing at the right end of the line. In order to correct this overslant, some writers turn the hand and wrist over to the left, as it moves along the line, so that the palm is more turned-down or prone and the penholder is more nearly vertical. This move-

ment is called "pronation." The reader may experiment by comparing one series of strokes made while the palm is prone and another in which the hand is turned over toward the right, and he will see that those

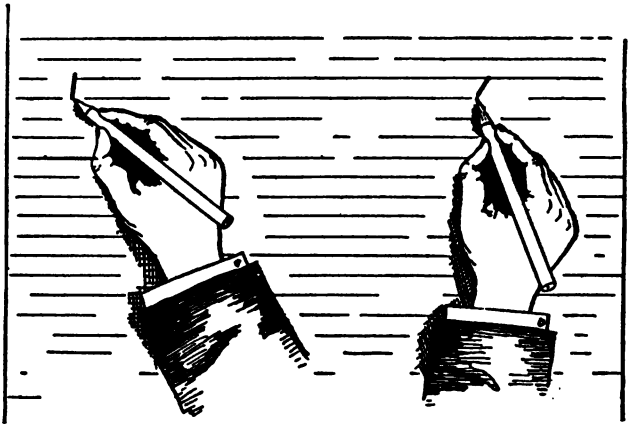


FIG. 1. CHANGE IN SLANT DUE TO THE MOVEMENT OF THE HAND ALONG THE LINE

made with the hand turned down are more nearly vertical than the others. Turning the hand down in pronation, therefore, tends to correct the overslant of the writing at the end of the line.

There are also adjustments of the eyes and of the body. It will thus be seen that all the chief parts of the hand and arm are used in writing, but this does not exhaust the list of movements which are made or of adjustments which must be maintained. The eye moves in order to follow the writing of the letters and to guide the movement. This must not be regarded as

an unimportant part of the whole activity. It is so important that it has been largely due to the recognition of the eye movements that important changes in the teaching of writing have been made. In order that the arm may be free to move and may have a solid support, the body must be held in position. The position which is taken will have an important effect upon the ease and fluency of the movement. This also has an important bearing upon the practical determination of the type of writing which ought to be taught.

Health demands good posture. The correct posture in writing is also determined by the demands of health. In order that the eyes shall not be subjected to strain, the writer must face the desk squarely and the paper must be near the middle line of the body so as to bring the two eyes the same distance from the writing. In order that the spine shall not be twisted and bent to the side, both arms should rest on the desk. In order that the lungs, stomach, and other vital organs shall have sufficient room, the body should be erect.

The application of correct posture to the position of arm and paper. It was formerly thought that these conditions necessitated vertical writing. Such is not the case, but it is necessary that the body or the head should not be turned or twisted. The paper may be tilted about thirty degrees to the left so that the line on the page is about at right angles to the line of the forearm. This will enable the arm to carry the hand along the line. The writing will then naturally have

a slant from the perpendicular by about the amount to which the paper is tilted, as is shown in Figure 2. This is based upon the fact that the upward and downward strokes of the writing naturally fall approximately in the direction perpendicular to the edge of the desk. Writing which has an angle of sixty to seventy degrees to the base-line is, then, the best suited to meet the requirements for an easy movement and at the same time to satisfy the demands of hygiene. More particular specifications of method must be left to discussions of the teaching of writing.

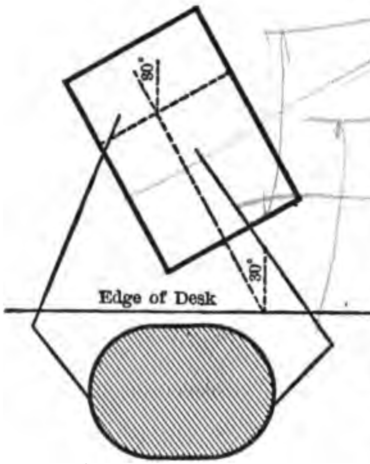


FIG. 2. DIAGRAM OF THE RELATION OF THE BODY AND ARMS TO THE DESK AND THE PAPER

The first process in motor learning is the selection of the appropriate movements. When the child first attempts to write, he is deficient in that he is not able to select just the movements which will be appropriate to produce the desired result. The first point, then, concerns the selection of the appropriate movements. The child by the time he begins to learn to write has sufficient general control over his bodily activities so

that he can in a general way call into action the groups of muscles which are to be used in such an act as writing. If he sees somebody else write, he can imitate very roughly the manner of holding the pen and of making the movements. But his attempt does not result in the contraction of just the muscles which produced particular strokes. Many useless movements are made. It may be seen from the observation of the child in the earlier stages of his learning that this extension to unnecessary movements is widespread. He contracts the muscles of the face, twists the body into unnatural positions, perhaps clenches the left hand, and presents the spectacle of general muscular tension. This preliminary stage in the development of movement is called "diffusion." The nervous energy has as yet no well-connected channels of discharge, and therefore scatters over a large area of the nervous system. Learning consists, then, in the first place, in confining the nervous energy, so far as possible, to the particular muscles which are needed to produce the effect.

Selection means also inhibition. This selection of the appropriate movements or the reduction of diffusion involves the inhibition [1] or the prevention of the contrary movements. There are two possible methods of attempting this selection. We may think particu-

[1] Inhibition means the checking or prevention of any activity, whether movement or train of thought. Thus the thought of the coldness of the water may inhibit the impulse to jump in, or the contraction of the muscles which raise the foot in walking inhibit the contraction of the muscles which extend it.

larly of the movement which is to be made or we may fix our mind upon the movements which are to be inhibited. In some cases, it is not well to think of the movements at all, but only of the result which is to be obtained by them, in this case the formation of the letters on the paper. Whichever object is in the attention, the same principle holds. This principle is that it is better to fix the attention upon the movements which are to be made or upon the result which is desired, rather than upon the movements which are to be eliminated or the result which is not desired. A stock illustration of this principle may be taken from learning to ride a bicycle. When a person thinks of the obstruction in the road or of the ditch at the side, he is very likely to run into the obstruction or fall into the ditch. His salvation consists in keeping his mind fixed upon the path along which he wishes to travel.

Application. The application of this principle to writing is that the child should not be made anxious concerning the useless or superfluous movements which he makes. In its application to the form of the letters, he should be thinking chiefly of the form which he wishes to produce rather than of those forms which he wishes to avoid. It is, of course, necessary for him to examine his own writing so that he may detect the faults in it in contrast with the correctly formed letters. He is apt, if this is not done, to fail entirely to see the faults in his writing; but after the fault has been detected, his attention is then to be fixed on the cor-

rect form which is to be substituted for the incorrect one.

Along with the selection of the appropriate movements goes the organization of these separate movements into the total or combined movements. This building together of movements is called "coördination."[1] In a coördination the movements must be adjusted to one another in such a way that each one is made with the proper amount of force. The failure to have them properly adjusted may be seen from the results of an experiment in which the pressure exerted by the thumb and the two fingers was measured. It was found that in a straight downward stroke the index finger, as one should expect, exerts the chief pressure, while the second finger and the thumb guide the movement. If, now, the downward stroke curves to the left, it is because the middle finger exerts an undue amount of pressure. If each finger does not exert its due or proportional amount of force in making any particular line, the stroke is thrown out of its correct course.

The successive movements must also follow one another in the proper time. The smoothness and ease of movement will depend very largely upon whether or not the successive parts thus follow one another as they should. There are certain gymnastic feats, for

[1] A motor coördination is a group of simple movements which combine simultaneously or successively to produce some definite result.

example, which are not difficult, provided one makes the movement or exerts the effort at the proper time, but which are impossible if this is not done. A simple illustration of the failure to time the writing movements correctly is taken from the relationship of the movements of the arm in carrying the hand across the page and the movements of the fingers in forming the letters. Some writers alternate these two movements. They use the fingers extensively in the formation of several letters and then interrupt the finger movement to lift the hand and carry it forward. A much better relationship is maintained between these movements by performing them simultaneously.

Rhythm is an important aid in timing the movements, The best means of bringing about the correct timing of the component movements of a coördination is to make the whole movement rhythmical. When the movement is carried on so that certain prominent strokes are made at regular intervals of time, as in marching to music, all of the elements of the movement are apt to fall into place at the proper time and with the proper force. This principle is used in writing when a class writes to count or to music It has been found by experiment that the use of rhythm which is suited to the age and training of the pupils increases the speed and improves the quality of their writing. If too rapid a rate is used, the quality becomes poorer. Not all the children of a class can profitably use the rate which is suited to the majority and a few are

deficient in the sense of rhythm, but if due provision is made for the exceptional cases the use of rhythm is a valuable means of teaching writing, or in fact any form of motor coördination.

A movement is learned through repetition. The method by which the coördination is formed through the selection of movements and their organization is the frequent repetition of trials. The coördination cannot be formed by telling the pupil how the movement is made. It is necessary that he find out for himself through making an attempt, noting the results of his attempt, and then endeavoring to improve his movement so as to retain these results which are the ones aimed at and eliminate those which are not the ones desired. This method of learning is sometimes called the "trial-and-error" method and sometimes the "trial-and-success" method,[1] according as we are thinking of the elimination of the wrong trials or the retaining of the correct ones. The fact cannot be too strongly emphasized that the only method by which such a form of learning can succeed is through practice. The child may be assisted in taking the correct position by being instructed, or being shown how, and his attention may be directed toward his mistakes. In a variety of ways verbal instruction as to how the move-

[1] The trial-and-error, or trial-and-success, method of learning is one in which one learns to make an adjustment, not by studying the problem which is presented, but by attacking it more or less blindly and then gradually eliminating the unsuccessful, and retaining the successful trials.

ment should be made is of great value to the learner, but it only supplements the trials which he makes and can never be a substitute for them. Moreover, it is necessary that *frequent* trials be made. The mistake is sometimes made, though not so often as formerly, of requiring the pupil to make a few copies only at each practice period and to make each one slowly and laboriously. A much better method is to require him to make a large number of copies somewhat more fluently, endeavoring to improve in each case over his preceding trial.

As learning progresses, attention is freed from the details of the movement. As the child learns better to coördinate the movements, a change takes place in his mental attitude toward his bodily response. At the beginning it was necessary to think of each separate movement and each stage in the making of a letter. As he becomes more skilled, he is able to make a series of movements or to form a letter or a word, thinking only of the whole group of movements or of strokes. The recognition of the movement becomes simplified and the details are given over to the control of the nervous centers which produce the movement. When this simplification of the recognition is carried far enough, we reach the automatic stage of the habit. This means that the attention is turned to the thought which is being expressed.

Increased skill makes possible the development of rhythm. As the child's attention becomes freed from

the details of the movements or of the letters, he is able to recognize more fully some of the more general features of the writing. One of these general features which has already been mentioned is rhythm. The child can write rhythmically only when he does not have to stop to consider how to form the details of the letters or to adjust his hand or fingers. Rhythm appears when the successive strokes are made in about the same time. Measurements have shown that this is true of the writing of the older child or the adult, even though the successive strokes differ greatly in length. The age at which the writing becomes thus markedly rhythmical in the case of the majority of children is nine or ten. We may conclude from this fact that this is the age at which it is suitable to give the child writing drills which require a high degree of rhythm.

Changes in speed within the letter are more determined by the mechanical ease or difficulty in adult than in children's writing. Another significant fact which has been discovered by experiment and which indicates that the child's attention is being withdrawn from the details of the letters is that in the writing of the older child and the adult the speed varies much more in the different parts of the letter than in the writing of the younger child. Where the mechanical difficulties are considerable, as where there is an abrupt change in direction, the movement continues to be relatively slow, while where the mechanical diffi-

culties are slight, as on a long, sweeping stroke, the speed increases.

Attention can be given to uniformity. Another general feature of the writing besides rhythm to which the child can give attention after the details cease to trouble him is the uniformity of the words and letters. This includes uniformity of slant, of alinement or of spacing. Young children commonly form their letters carefully, but their writing presents an uneven appearance.

The writing habit includes word and letter recognition as well as movement. We are accustomed to think of such a habit as writing as being composed of movements. This is true, but movements are not all there is to the habit. The movements are made in response to the sight of letters and words. At the very beginning the writer looks at the letters and copies them, thus holding the form of the letters in mind for only a short time. Later on, the letters need not be seen, but must be present in imagination. Later still, the form of the letters may largely drop out of mind, but even in this most advanced stage the thought of the word exists in the mind and is the stimulus to the movement. In the case of writing, then, we may separate two phases of habit, that of the movement and of the recognition of the stimulus.

Writing stimulates a study of letter forms. It is appropriate to say in this connection that the writing habit, considered as a form of movement, is of import-

ance in the development of the recognition of the letters. Before the child begins to write, it is necessary that he shall be able to distinguish the forms of the different letters roughly. He may, however, distinguish them sufficiently for reading, but not for writing. As he tries to reproduce the forms which he perceives, he finds it necessary to pay more attention to the form than he has previously done. Writing furnishes a stimulus to a further study of the form of the letters.

Writing also furnishes an additional experience which enriches the perception of form. After a person has learned to write letters, his recognition of them is not merely a matter of sight. There is a tendency, when the letters are seen, to make the movement of producing them. There are numerous illustrations from our mental life which show that our recognition of objects includes the experience which we have had in handling them. If we lift two boxes which are the same in weight, but of different size, we are subject to an illusion. The smaller box appears to be much heavier than the large one. This is because in looking at the boxes the sensation of sight produces an anticipatory contraction of the muscles which would be used in lifting the boxes. When, now, we do lift them, there is a strong contrast between the amount of force which is exerted in the case of the two hands, because we had anticipated the necessity of lifting the larger box more vigorously. This contrast causes the feeling of greater strain in lifting the small box which makes

it appear heavier than the larger one. So also in our rceognition of letter forms there is included something of the motor experience of making them.

The writing movement becomes the means of expressing meanings. We have been discussing writing as a form of movement which becomes organized so that it becomes an effective means of producing the forms of letters and words on paper. The true function of writing is fulfilled when, in addition to this, it becomes the expression of meanings. Meanings are represented first in spoken words, and when we write we commonly say over and hear inwardly the words. Because meaning was originally associated with the spoken and heard word, it is still embodied in internal speaking and hearing. When writing advances to the stage at which it is a ready means of expressing meanings, it is carried on as a direct response to ideas embodied in these forms. The consciousness of the movements or of the details of the form of the letters recedes to the background. When this stage is reached writing has become automatic.

The habit may become automatic too soon. In the development of writing toward this automatic[1] stage, there may be two opposite faults. The habit may become automatic at a stage when it is not sufficiently well developed; that is, the person may learn to express his thoughts through a form of movement which is

[1] An activity is automatic when it is carried on with little or no attention to the activity itself. Thus one may take a key from his pocket and unlock a door automatically.

irregular and which produces illegible letters. The cure of this fault is to give attention to the habit until it becomes developed to the point at which the results of the writing are satisfactory. The results may be unsatisfactory either because the movement is awkward or because the writing is poor in quality.

Attention may be given to the details too long. The opposite fault also sometimes occurs. In this case a person never loses his consciousness of the writing movement or of the form of the letters which he is producing sufficiently to give his attention wholly or chiefly to the thought. Persons sometimes possess two styles of writing, one in which the habit is automatic and the writing is very poor, and another in which the writing is kept at a high grade through giving attention to it. This development of two sorts of habit, one which is used in the ready expression of thought and the other in producing pleasing forms, is unfortunate. The child should develop his writing habit to a sufficient degree of perfection that he may produce good results, but it should then become automatic.

Movement and form both require attention. Different systems of teaching give different emphasis to the development of a good motor coördination as compared with the production of writing which presents a good appearance. Some teachers believe that if care is taken to develop good movement the form will take care of itself, or at least will not need much attention. It is true that writing is to be thought of as a

movement habit, and that the principles which govern its development as a movement are those which must guide the teacher in the methods which he uses, as we have already seen in detail. But a movement is made in response to a stimulus, and it is to be judged by the accuracy with which it is adjusted to the stimulus. A movement is not judged merely according to its ease or smoothness. The handwriting movement, accordingly, may be smooth and easy, and yet the writing may not be legible because the letters are poorly formed or not well spaced.

The child should study form. There are effective and ineffective ways of calling the child's attention to the form of the letters which he is producing. One method is merely to place a model before him and require him to copy it. This method, if used alone, is seriously deficient in two ways. In the first place, it gives the child merely the finished product, and, if the copy is engraved, a finished product that he could not possibly imitate. In place of this copy he should have an opportunity to see the form *being produced.* The form does not in itself suggest the best method of reproducing it. The form must be recognized by the child in terms of the movement of producing it. In the second place, merely to set before the child a perfect model does not enable him to see just in what respect his writing differs from it. To gain this end it is necessary to show him how to make an analysis of the form of his writing.

Form should be analyzed. One means by which the child may be helped to make such an analysis is to show him series of charts in which progressive degrees of excellence in each of the main elements of form are illustrated.[1] For example, one series of charts may show different degrees of uniformity in slant or alinement, another different qualities of line or stroke, another different degrees of excellence in letter formation, and another degrees of excellence in spacing. By the study of such charts and an analysis of his own writing the child may gain a clearer recognition of the ways in which his writing deviates from the model.

The child can be guided in the development of the best movement. If some teachers emphasize movement to the neglect of form, others commit the opposite error. It is clear from the whole trend of our discussion why this is a mistake. It is a far safer and more economical method to guide the child in the formation of those habits of movement which experience has shown to be the most efficient than to allow him to blunder into a form of movement without any assistance. Substantially the same form may be produced in a variety of ways, and some of these are much better than others. It is the business of the teacher to help the child to find these better ways as rapidly as possible.

[1] Such a series is published in the appendix of *The Teaching of Handwriting*, by the author, published by Houghton Mifflin Company (1914).

The elements which can be acquired through instruction constitute good form. It has been said in a former paragraph that the writing movement could not be learned through instruction, but must be developed through the trial and success method. This applies to what may be called the execution of the movement. There are certain aspects of the movement, on the other hand, which may be partly learned through imitation or through instruction. The child may gain help through these methods in learning how to hold the pen, what position should be assumed by the hand and arm, and in a general way what parts to move in making the strokes. These aspects of the activity may be called "form." Form, in this sense, is, of course, entirely different from the form of the letters or the words. We use the term in the same sense as when we speak of good form in playing tennis or performing a gymnastic feat or riding a horse. There are elements of good form in writing which there is not space to enumerate here. A general idea of what they are may be gained from the illustrations in Figures 3 and 4. The acquirement of good form adds considerably to the ease and accuracy of the writing movement, or of any other activity.

Speed and quality should progress together. Another question of method is related to this issue between movement and form. Those who emphasize movement to the exclusion of form often require the child to write very rapidly from the start, while those

FIG. 3. CORRECT POSITIONS OF BODY, AND ADMISSIBLE POSITIONS OF THE HAND FOR YOUNG CHILDREN

Body position from C. P. Zaner, *Zaner Method*, Manual no. 1. Hand positions from W. A. Whitehouse, *The Modern Writing Master*, vol. I, no. 1. (With the permission of the publishers.)

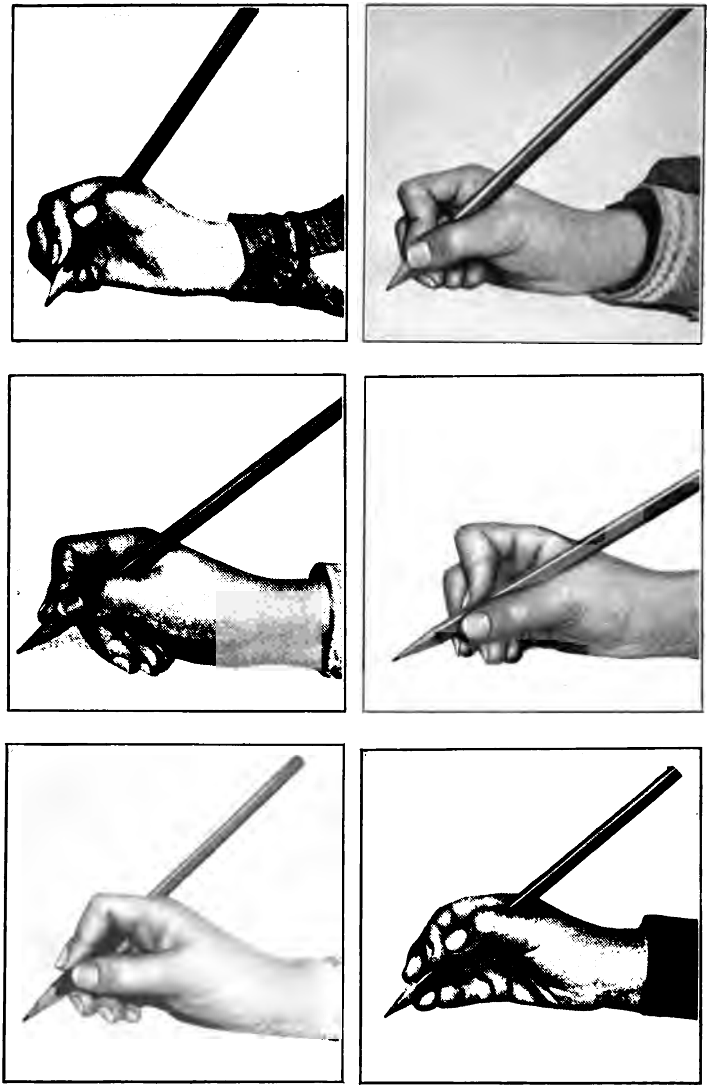


FIG. 4. INCORRECT POSITIONS OF THE HAND

From W. A. Whitehouse, *The Modern Writing Master*, vol. I, no. 1. (With the permission of the publisher.)

who emphasize form chiefly pay little attention to speed, particularly at the beginning. The practice of allowing the child to write excessively slowly was criticized above on the ground that it does not give the pupil sufficient practice. It may also be criticized on the ground that it does not give the pupil practice in connecting the elements of the movement to make a continuous process. When the child makes the letters by a slow, drawing movement, he is not obtaining a sensation of the whole movement in such a way that he learns how it feels to write a letter or a word. On the other hand, if the child is required at the beginning to make the movement as rapidly as he will later, it is impossible for him to form the letters correctly. As a consequence he is not learning to make the movements which he will later have to make. He is also forming habits of carelessness and is failing to get a clear and correct idea of the form of the letters. The solution of this question, then, is that the child should progress in accuracy and form together.

The child should be able to trace his own progress. It follows, from this analysis of the qualities of good writing, that the child should be able to trace his progress in both quality and speed. The more definite his attainment can be made, the more stimulus he will receive toward improvement. The common method of grading the child's attainments by comparing him with others in a rough, general way is very deficient, because it does not give him a sufficiently definite idea

either of the degree of skill which he has attained or the direction in which his improvement should proceed. The child should, then, be compared with his own past record rather than with that of other pupils, and this should be expressed in a form as definite as possible. The speed may be measured by occasional tests and the quality by analyzing it by the method which was suggested above.

The correct habit should be practiced in all writing. We have already seen that a person's writing habit may be of one sort when he is writing for the sake of good form and of another sort when he is writing to express thought. The child should so develop his writing habit that much the same result appears in the ordinary work as in the writing lesson. In order that this may be the case, it is necessary that some attention be paid to the writing which he produces in the rest of his school work. It would be well to do this throughout the latter part of the elementary school. In the first three or four grades the child's attention should not be distracted from the thought which he is expressing by the necessity of thinking much about the quality of the writing. He has great difficulty at the best in expressing his thoughts in writing at this time, since the unfamiliar habit distracts his mind; but in the later grades it will not distract the child to exercise a little care in order that the writing shall not deteriorate. It is well, then, to give the handwriting grade upon the quality of the work which is turned

out in the other subjects as well as that which is produced in the writing lesson. In fact, this should probably be the main stimulus to progress in writing in the upper grades.

Modifications due to age

Motor ability changes with age. What has been said indicates that it is necessary to modify the method of teaching writing to suit the age and stage of development of the pupil. Some methods of teaching writing which are in wide use take no account of the difference in ability of the child at different ages. For example, the same speed in writing is required of the first-grade child as of the eighth-grade child. Studies of the motor ability of children at different ages indicate that there is a marked increase in the rapidity of movement and in the steadiness and precision of movement from the age of six to that of sixteen. A study of the ability of children in different grades in making a series of upward and downward strokes as rapidly as possible demonstrates the same fact. A survey of the rapidity of writing, in all the grades from the second to the eighth among over thirty thousand children, shows that on the average the first-grade pupil writes less than half as fast as the eighth-grade pupil.

The fundamental and accessory movement theory. The difference in ability between the younger and the older child has sometimes been ascribed to the devel-

opment from fundamental to accessory movements.[1] The theory is that the younger child is able to make the so-called fundamental movements more readily than the accessory movements, and that improvement in ability is chiefly in gaining control over these accessory movements. The distinction between the fundamental and accessory movements is not always clear. The fundamental movements are sometimes described as those which are old in the race, such as walking; while the accessory movements are those which have been acquired in human life, such as manipulation. Sometimes the distinction is made between the movements of the large muscles, or large movements, which are designated as "fundamental," and the opposite kind of movements which are called "accessory." Finally fundamental movements are sometimes described as "central," that is, concerned with the trunk or with the parts of the limbs next to the trunk, while accessory movements are those toward the extremities. An examination of the movements which are made by the young child will indicate that this distinction is not a very useful one. In the problem of learning we are not concerned with instinctive movements, and other movements do not develop in any consistent way from central to peripheral or from large to small.

The suitability of movements for the child should be

[1] A fundamental movement is one which is old in the history of the race, is a large movement or is made with large muscles, and is central (in or near the trunk). An accessory movement is the opposite.

judged on the score of their rapidity, precision, and complexity. A much more valid distinction between the movements which are suited to the younger child and the older child rests on the facts which have already been referred to. The young child is not capable of very rapid or precise movements. To this may be added the further fact that movements are difficult for him in proportion to their complexity. The combination of a large number of movements in one complicated coördination presents difficulties to the child in proportion to his deficiency in motor control.

The difficulties should be lightened for the young child. These characteristics of the child make the acquirement of handwriting a difficult matter for him and one which is attended with considerable nervous strain. Care must be taken, therefore, to ameliorate for him the difficulty of the task. This may be done by making the requirements for accuracy and rapidity low and by placing the handwriting period at a time when the child is not fatigued. The requirement for speed has already been discussed. The avoidance of undue accuracy of form may be reached by allowing the child to write very large. At the beginning the writing should be done on the blackboard. When the child first uses paper, he should write with large letters and with a pencil which offers little difficulty in its manipulation. The pencil should be fairly large and the lead smooth. The paper should also have such a surface that the mark is easily made upon it. As the

child grows older and gains in skill, the writing may be gradually decreased in size and may become gradually more precise.

Individual differences and practice periods

Individual differences. Besides these differences due to age, there are wide differences between individual children in this as in other forms of ability. Some children are superior in their writing ability to the average of several grades above them, whereas others are equal only to the average of several grades below. These individual differences may be taken account of in several ways. The ideal method would be to group children according to their ability in each separate kind of work, so that those of like ability are together and that those who have reached a certain standard might be relieved of further work in that subject. This, however, is not possible so long as the rigid grade system maintains, and the best that can be done is to recognize the differences among children of the same grade and to adjust the demand made upon the different children according to their various abilities. As was suggested above, each child should be judged on the basis of his past performances instead of by comparison with his classmates. In the speed of writing it is very desirable to have as many of the children as can comfortably do so engage in rhythmic exercises to music or to time marked in some other manner, but it is necessary that the children who do not naturally fit in with this rhythm should not be required to follow it.

The length of practice periods. In arranging the conditions so that the practice will be most effective in producing improvement, it is necessary to take account of the principles of economy in effort due to the arrangement of the periods of practice. It has been found by experimental investigation that the length of period or the interval between the periods of practice affects the rapidity of learning. In the case of the development of a motor habit, such as handwriting, the same amount of time split up into rather short periods is more effective than if it is all expended in long periods. With the child in the earlier grades ten minutes is probably the best length of period, and in the upper grades not over half an hour can be spent to the greatest advantage.

REFERENCES

1. L. P. Ayers: *Scale for Measuring the Quality of Handwriting of School Children*. Bulletin no. 113, Division of Education, Russell Sage Foundation.
2. L. P. Ayers: *Scale for Measuring the Quality of Handwriting of Adults*. Bulletin no. E138, Division of Education, Russell Sage Foundation.
3. Frank N. Freeman: *The Teaching of Handwriting*. Houghton Mifflin Company. 1914.
4. Frank N. Freeman: *An Experimental Study of Handwriting. Psychological Monographs*. 1914.
5. Charles H. Judd: *Genetic Psychology for Teachers*. D. Appleton & Co. (1903), pp. 84–97 and chap. VI.
6. S. H. Rowe: *Habit Formation and the Science of Teaching*. Longmans, Green & Co. 1909.
7. E. L. Thorndike: "Handwriting." *Teachers College Record*, Columbia University. 1910.
8. W. A. Whitehouse: "Correct Penholding." *The Modern Writing-Master*. Boston, Mass. (1906), 1, 24–37.

CHAPTER III

DRAWING: PERCEPTUAL LEARNING

In learning to write, the child develops habits of movement and acquires the recognition of form, but since the motor coördination is the more prominent element, writing is taken as an illustration of sensori-motor learning. Drawing also includes both the recognition of form and its representation, but in this case the element of recognition is more prominent, and therefore drawing is taken as an illustration of perceptual learning. Drawing is commonly regarded as an illustration of the development of motor habits. The production of the form upon the paper is thought of as the chief element in the process. This, however, is a secondary part of the whole process as will be made clear by the following illustration.

A demonstration experiment in drawing

In order that the reader may have in his experience a concrete illustration of the development of the recognition of form, the following simple experiment may be performed. Upon page 36 will be found a figure (Figure 5) which should not be looked at until the directions given below have been read and clearly understood. The figure is to be observed for the brief

interval of ten seconds, after which the attempt should be made to reproduce it. During the time the drawing is being reproduced, the book should be turned over so that it cannot be seen. After the figure has been completed as well as possible, the paper should be folded so as to conceal the drawing and another attempt made in the same manner. This process should be repeated until the experimenter is satisfied that the drawing is reproduced substantially in its correct form. It is not necessary that it shall be minutely accurate, but only that the chief elements shall be present in about their true relations.

The act of drawing has produced completer recognition. After the drawing has been made, the reader should analyze his experience and test the correctness of the following account. The first question for examination concerns the difference between the perception of the figure after the figure has been drawn in the final trial in comparison with the type of recognition which existed before it had been mastered. It is clear that, as a result of the study and the attempt to reproduce the figure, something has taken place which materially modifies the recognition of it. This has not been in the nature of mental development in general, but has rather consisted in the development of the ability to recognize this particular figure. Before the study, the figure was seen in a vague, indefinite manner. It was noticed that there were a number of lines, but the shape of the lines or their exact relationship

to each other was seen only very vaguely. In this first preliminary recognition some persons emphasize more the general outline of the figure and others recognize one or two details clearly, but lose sight of the rest. Whichever type one belongs to, after the study of the figure, one is able to grasp the general outline and at the same time the details of which it is composed.

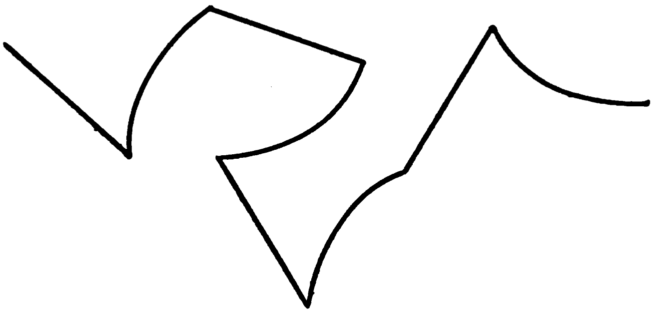


FIG. 5

Completer recognition is gained through active exploration. The next question to be considered is the means by which the development has been attained which results in this different manner of recognition. In the great majority of cases, when the question is raised, it will be found that one has begun the study of the figure at a particular point, usually the left end of the figure. It is instructive to ask ourselves why most people begin at the left end rather than at the right end. The explanation is undoubtedly to be found in the fact that we have learned to recognize words on

a printed page in this order and that we have learned to write from left to right. This means, then, that we bring to the study of this figure certain habits of exploration which determine the kind of procedure we shall take.

General ideas are employed. There are also other features of the process of studying the figure which indicate the existence of a definite series of mental habits based upon our past experience. For example, the number of lines was very likely determined by counting. In this we use the general idea of number or the habit of thinking of things by number as instruments in developing the recognition of the figure. Very likely also the kind of lines was noted and it was seen that there were some straight lines and some curved lines. The alternation of the straight and curved lines was likely also to have been observed, and the fact that the lines formed angles with one another and that these angles were perhaps of about so many degrees. Very often the figure is grasped by comparing it with familiar objects, such as a letter of the alphabet, or stenographic signs.

The nature of perception as illustrated in drawing

Development of perception consists in organizing impressions, not in gaining new ones.[1] This description of the way in which we set about such a task will show

[1] Formal definitions of perception and sensation are given on p. 51.

that we bring to it a vast number of ideas and habits of procedure out of our past experience and that our mode of gaining a mastery of the figure is not passively to allow it to make an impression on the mind, but rather actively to go about its study, using these ideas and habits of procedure. We may sum up this experience by saying that the recognition of such an object is not chiefly a matter of gaining impressions, but rather a matter of actively analyzing the characteristics of the object. Put in another way, this principle means that the development of the ability to recognize such an object is not primarily the development of keenness of the senses, but rather the development of a mode of arranging or organizing the sensations which we receive from the object and giving to them a meaning.

Such organization includes analysis and synthesis. The analysis of such a figure into its parts makes possible the grasp of the figure as a whole. In the final recognition we were not merely able to see each part clearly, but we were able to grasp a larger number of the parts because each one had previously been studied for itself. The whole process consists of the clearer recognition of the parts and better organization of the parts in relation to one another. Put in technical terms, such recognition is based on both analysis and synthesis.

Example of the manner of directing the child in the development of his perception. The fact that in order

to gain a clear perception of an object it is necessary to go about the study of it in an active manner finds illustration in all the work of the school in which a study is made of concrete objects. It gives the clue to the manner in which the teacher can guide the child's efforts to gain an accurate and comprehensive notion of such an object as a plant or an animal. The child should not be allowed merely to get a vague impression by passively looking at the object, but he should be led, through questions and discussion, to examine the various parts in detail and to discover their likenesses to the parts of other similar objects or their differences from them. Thus, in studying a cow the child should be induced to examine its teeth, to compare them with the teeth of the sheep, the horse, the dog, cat, etc. These facts are then to be related to the different kinds of food which are eaten by the different classes of animals. In similar manner the child examines hoofs, horns, etc. As he gains a clearer idea of these details he is also at the same time putting them together so that they combine to give him a complete notion of the animal as a whole.

Past experience is involved in all perception. These characteristics are true of our perception in general. One of the most prominent characteristics of perception is that we depend for our recognition of things in large measure upon the result of our past experience. The bearing of our past experience upon our present recognition and responses is easily recognized in cases

in which the differences due to differences in past experience are more obvious. In such cases the activity of past experience in governing interpretation is called "apperception." Many objects have a widely different meaning to different persons because of their past training or education. The artist sees in a rural scene the beauty of the landscape; the farmer, on the other hand, may see the possibility of raising crops; the huntsman may think of it as containing game; and so on.

All perception is apperception. The bearing of past experience in such cases is obvious, but it is equally present in every recognition. The setting-off of a few of our experiences as being characterized by apperception[1] as distinguished from others which we call "perception" obscures this fact. Apperception is present in every case of perception, and is therefore not any unusual or peculiar characteristic of mental life, but is rather present throughout its whole range.

Object teaching is not merely showing the child objects. The practical application of this principle is that different persons will give a different interpretation to their experiences because of the different background in which they are set. We recognize this in the case of the more conscious interpretations we make,

[1] Apperception is a broad term which designates the part played by the mind itself in determining the character of our experience. What our experience is depends on our inherited cast of mind and on our previous experience as well as upon the stimuli which affect us. This activity of the mind in coloring experience is apperception.

but it is also of great importance even in the perception of physical objects. The mistake is sometimes made of supposing that when we set up an object before a child his recognition of it is the same as ours. Object teaching does not merely consist in presenting things to the child, but rather consists in giving him such modes of interpretation that he can see in the object the thing which the educated person sees. Likeness in the stimulus does not guarantee likeness in the response. There must be also a similarity in the form of development of the two persons who are making the response.

Illustration of the need of past experiences as a background for the complete recognition of objects. The task of the school with reference to the child's recognition of the objects of the world about him is systematically to furnish him with experiences or use the experiences which he has so that he will have an adequate background for their interpretation. One of the common objects of the modern person's environment is the trolley car. To the majority of the adults of the present, who did not learn about electricity in their school days, the trolley car is a mystery. They know there is something called electricity, which travels along the trolley wire, and that when the trolley is in contact with the wire and the motorman does something with the handles before him the car starts. In order to see more than this in a trolley car one has to become familiar with the dynamo and electric motor, to learn how a wire passing through a magnetic

field generates an electric current, and that when a current passes through a wire which is in a magnetic field motion is generated. One thus comes to see that the electric current is a means by which the motion which is generated at the power-house through steam or water power is reproduced at the wheel of the trolley car. Thus we see that the complete recognition of a trolley car depends on having a great many preliminary experiences.

Drawing (with the other representative arts) selects particular features for representation. The influence of past experience upon the type of recognition has a particular illustration in drawing. Drawing is not a photographic representation of all that is seen. There is always, either consciously or unconsciously, a selection of features of the total appearance to be represented. The history of the development of the arts of drawing and painting indicates that those aspects of objects which are selected to be represented vary with the stage of progress which an art has reached. The ultra-modern schools, as the Cubists and Futurists, are attempting to develop new modes of representation which will depict aspects hitherto not represented, such as motion.

Representation by drawing depends on the mastery of a drawing language. Learning to draw, in fact, may be thought of as an acquirement of a language or a mode of expression. This language has gone through stages of development just as has spoken language.

The early Egyptians were able to reproduce certain characteristics of the form of objects, but lacked entirely the ability to represent perspective, or the differences in the distances of objects, by means of the lines of their drawing. One object was represented as being farther away than another merely by being partly hidden by it. We may speak of the Egyptian mode of representation, then, as one sort of dialect of the drawing language. We may also compare the art which has been developed in Oriental countries with that of our own, and we find here also two different dialects. Chinese and Japanese painting, for example, consists chiefly in the use of lines instead of the masses which are used in Western painting, and until recently Occidentals failed to appreciate the significance of these drawings.

One sees, in a measure, what one can represent. These differences in mode of representation are not merely differences in expression, but also represent different modes of seeing the object. A person who has had one type of training sees a particular aspect of the thing and represents that aspect because he has acquired the mode of representation which is suited to it. He has developed a tool of expression, and as a consequence he can see the things which are capable of representation by this tool.

In his early experience the child learns to recognize the real form from the appearance it presents. An example of the way in which drawing serves as a mo-

tive to seeing things in a different manner or in a more adequate manner may be taken from the development of the child. When the child begins to draw, he is not able to represent perspective by means of the lines and their relationship to one another. The reason for this is that he draws the objects which he sees as he knows them to be rather than as they appear from a particular point of view. He has learned to see the object as it really is and not to recognize the particular appearance which it presents as he looks at it. The child's form of recognition is one which he has bought at the expense of a good deal of time and experience. When the child first observes objects, he is undoubtedly impressed by their appearance, and he does not at first realize that an object which presents one shape from one point of observation and another from another point is really an object which has its own constant shape independently of the position from which it is viewed. He learns this lesson thoroughly. For example, he knows that the top of a table is rectangular in spite of the fact that it appears from most positions to have acute and obtuse angles. When now he draws a table, he represents it in this rectangular shape.

Drawing is the representation of the appearance. The difficulty in learning to represent perspective, — that is, to draw the object as it appears from a particular point of view, — is that we are incapable at first of clearly recognizing how the object does appear. One must, then, learn to give attention to the appearance

in order that he may reproduce this on paper, and thus give the impression which the object makes upon him.

The recognition of the appearance comes from trying to draw it. The child acquires this ability to pay attention to the aspect of an object which is presented from a particular point of view only through the acquirement of technique of expression. Merely to lecture to the child upon perspective or to discuss the principle of vanishing points, horizon, and so on, as was formerly done, does not give him the form of recognition which he needs. The best method of doing this is to lead the child to experiment in putting lines in different relationships to one another so that they shall correctly represent the object which is being portrayed. For this purpose the three lines which represent the three dimensions of an object may serve as the basis of experimentation. After the child has learned to place these lines in such a way that they represent the appearance of a chair, say, or of a table, he has acquired an added ability in seeing as well as a new mode of expression.[1]

Illustration of the bearing of activity on perception. As the recognition of form is made accurate and complete by the active response of drawing, so also do the active responses which we make influence very greatly all our perceptions. We see and hear and touch in

[1] This is the method pursued in the University Elementary School of the University of Chicago under the direction of Professor Walter Sargent.

order that we may effectively move and handle and use. Sensations exist to guide actions. Therefore, the natural and most effective stimulus to perception is a demand for action toward the object in question. A boy who has a chance to run an engine or an automobile will soon find out enough about it for his practical purposes, and the practical impetus which he thus gains may carry him a good deal farther in his investigations. Not only so, but the experience he has gained in actually handling the machine gives him a far better understanding and basis for further knowledge than merely looking at it would have given. In a similar way the girl is led to a far more intimate and exact knowledge of food materials and cooking processes by a little practical experience, accompanied by the appropriate theoretical study, than by theoretical study alone.

Development of drawing with age

Children develop chiefly in the comprehension of complex forms. The ability of the child to represent objects varies to some extent with the stage of his development. The stage of development is based to a small degree upon the skill with which the pencil can be handled, but also to a much greater degree upon the extent to which the child can analyze an object and can comprehend the different parts of it in relation to one another. A test which has been made of school children of different ages indicates that the younger

and older children do not differ greatly in their ability to reproduce simple forms.[1] The individual differences among children in this respect are more prominent than are the differences between children of different ages. The suitability of a figure to the age or stage of development of the child would seem to be based more on its complexity than upon the accuracy with which a particular part must be observed or reproduced.

The young child's drawing is symbolic. Studies of the drawings of children indicate that there is a marked difference in the degree to which a child delineates the individual characteristics of an object, or the details of which it is composed. The child neglects the characteristics which distinguish an object from others of the same kind and represents only such features as enable another person to know the general class to which the object belongs. We express this fact by saying that the drawing of a child is symbolic. At about nine years of age the child seems to recognize more clearly the need of more accurate and detailed representation. An illustration of this transition appears in the fact that before this age the majority of children draw a face from the front view, whereas after this age the majority draw it from the side view. The side view is a much easier one to represent, and when the child endeavors to represent accurately, he chooses the easier mode.

[1] From an unpublished master's dissertation by S. P. Chinnappa, on file in the library of the University of Chicago.

Pictorial representation develops rapidly from the transition stage to adolescence. The two elements of pictorial representation which indicate its progress are the increase in the fullness and accuracy with which the particular characteristics of the individual objects are presented, as distinguished from the presentation of the characteristics merely which belong to the general class of which the object is a member, and the representation of perspective. The portrayal of perspective may be regarded as an artificial performance in the sense that it requires a type of attention to the appearance of objects which is not produced by the ordinary demands of life, but is due to the special demand of representing relations in three dimensions on a two-dimension surface. That this is counter to the usual type of perception is shown by the fact that it took the race so long to develop it. Early Egyptian art is almost completely lacking in perspective, the Greeks attacked the problem but did not solve it, and it was left for the painters of the Italian Renaissance finally to develop the true representation of depth by drawing. It is not surprising, therefore, that the child does not represent perspective in his early drawing, and its development is to be looked upon as dependent upon the acquirement of the intellectual capacity necessary to enable him to understand and adopt the methods which have been handed down rather than as a spontaneous growth.

Mechanical drawing and diagramming represent

structure and relations of parts. Mechanical drawing and diagramming more closely resemble in purpose the drawing of the young child than they do pictorial drawing. The distinction between diagramming and pictorial drawing is brought out sharply in an investigation, as yet unpublished, by Dr. Frederick C. Ayer. The subject of Ayer's study was the relation of drawing to the work in science. He first found that there was little relation between the student's ability in pictorial drawing and his rank in science work. He then studied the correlation between the ability to draw a diagram — for example, of the minuter structure of a feather, as seen under a low-power microscope — and ability in science. Diagramming calls for the analysis of the object into its parts and the comprehension of the relation of these parts. The drawing does not need to look like the object so long as it gives the observer the idea of the relation. Similarly, mechanical drawing does not give a realistic picture of the object. A plan drawing or an elevation represent only one face of an object, and often internal structure which cannot be seen at all; and in representing the parts which can be seen only shows enough to indicate the structure. Even a mechanical drawing in perspective is constructed according to rule, shows the object from a set point of view, and indicates the structure only. It can be made by any one who knows the rules and is careful in making his measurements and drawing his lines accurately. A comprehension of the structure of

the object and skill and accuracy in measurement and drawing are required, but not ability in recognizing relations in space as they appear to the eye, or the ability to reproduce these appearances.

Diagramming begins early, mechanical drawing at about adolescence. As has already been suggested, the child's early drawing is a crude sort of diagramming. His development in this type of drawing is dependent mainly upon the development of his ability to analyze objects and to understand their relations. Mechanical drawing, because it usually requires a conception of internal structure, and always accuracy in measurement and drawing, does not develop in any large degree until about adolescence.

Individuals differ in their rate of development and in the stage which is finally attained. While the development in accuracy, as represented by correctness of detail or by the degree to which perspective is shown, develops rapidly up to adolescence, there are individual differences in this respect, as in the others which have been mentioned. Very many children do not at adolescence equal the stage of development which others have attained several years before. In suiting the form of training to the child, then, the development which is characteristic of the average child must be taken into account, and also the different stages of development which different children represent at the same age.

General account of the nature of perception

In perception impressions or sensations are given an interpretation. The illustration of drawing has shown that the recognition of objects which are strange is not by any means a simple affair. When a child recognizes the form of a figure, as we have seen, he brings to bear upon the sensation which he receives through his eye the ideas and the habits of attention and of recognition which have been formed by his past experience. This application of past experience to our present impressions makes possible the interpretation of the sensations through which they acquire a meaning. When we receive a sense impression and add to it the results of other experiences in order to give it an interpretation, we call the mental process "perception."[1] In perception we usually bring to bear upon the impression the results of past impressions which have been received both through the same sense, and also through different senses. This whole process of recognition through the association of other experiences which we have had in the past may be illustrated by another example.

[1] A sensation is a simple experience which is produced by the stimulation of one of the sense organs; as, for example, the impression of a color, or a sound, or of a taste, or of a touch. Such a simple experience does not of itself have any meaning. A perception is a sensation which has acquired a meaning through its combination with other sensations. In perception there is the recognition of objects, while in sensation only a single quality is experienced. Sensation is an incomplete stage in the development of perception.

Illustration of the complexity of perception. We may take as an example the recognition of a cup of milk by the child. The impression which the child receives as he looks at the cup is one of sight, or of vision. He sees the color of the cup; he recognizes its outline, or its form. In addition to this, however, he thinks of the cup as having a certain degree of hardness; that is, he recognizes that if he should touch the cup it would offer resistance to his finger, and he does not attempt to compress it as he would a rubber ball. In this recognition he is applying the results of the previous experience gained through handling the object. Further, he recognizes that the cup has a certain weight. If he thinks of it as being full of milk, whereas in fact it is nearly empty, and if he then attempts to lift the cup, he will exert too much force. This may result in such application of force as to cause the milk to be spilled. This gives an evidence that there was in the recognition also a notion of the amount of force necessary to lift the object. The child also has some recognition of the sound which would be made if he were to knock the cup against some other object, and this expectation is illustrated when he strikes a spoon against the cup. If it gave forth a different sound from that which it usually gives he would exhibit surprise.

There is also a variety of present impressions. In addition to this obvious combination of the results of past experiences of different senses, there is also a

combination of different impressions received through the two eyes. It is a well-known fact of psychology that the recognition of the distance of objects depends on the fact that we get a slightly different view of an object through one eye from that which we get through the other. These two views are combined in our perception, and as a result of this fusion we recognize the distance. Furthermore, the fact that our eyes have to turn inward more to look at a near object than to look at a far one gives us an experience of muscular strain which we may use in distinguishing distances. When the child turns his head to look at the cup, he also gets sensations from the muscles and joints of the neck, which gives him a recognition of the direction of the object from his body.

Such a perception is a result of growth. This analysis of the various elements which go to make up such a simple perception as this is of no particular value merely as showing that the experience is a complicated one. Its value is rather in indicating that this perception must have had a growth or development; that it is not something which the child has merely as a result of the structure of his sense organs. Theories of perception have sometimes been held which assume that the object impressed itself upon the mind, or upon the sense organs. On the contrary, the recognition of the object is built up as a result of a variety of past experiences and through a combination of a number of different present elementary experiences.

Development is illustrated in overcoming illusions. We must not expect, then, that the recognition of the child is precisely the same as that of the older person. As it is a result of growth, so the stage of growth which has been reached in the child's mind is different from that which has been reached by the adult. Experiments have shown that recognition of objects by the adult is not perfect. The person who has had experience in drawing, for example, has a more complete

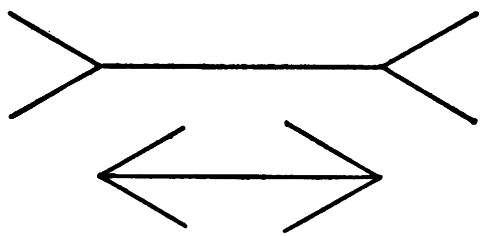


FIG. 6. MÜLLER-LYER ILLUSION

recognition of form than has the ordinary individual. Studies in optical illusions,[1] an example of which is shown in the accompanying Figure 6, indicate that while the ordinary person makes a large error in comparing the length of the lines in the recognition of such a figure, he may overcome the error by sufficient practice. After about one thousand trials one can judge correctly the relative length of the two lines.

[1] An illusion is a false or distorted perception. It is not commonly abnormal, but is a form of misinterpretation, common to nearly all persons, which can be explained by ordinary psychological principles.

Sensations at first become combined into perceptions in the recognition of an object which has practical meaning. One might conclude, from the analysis of perception which has been made, that the various sensations are combined in perception in some mechanical way or because the child has some inner tendency or other to build together sensations into perceptions. This would be an altogether false view of the matter. The sensations are built together as they contribute to the child's recognition of an *object*, and in the majority of cases, as has been already said, the child learns to recognize objects which have some practical value or meaning to him. The cup of milk may be taken again as an illustration. It is clear in this case that the chief motive of the child in paying attention to the cup and in learning to recognize its form, direction, distance, weight, etc., is the satisfaction of his appetite for the milk. Those sensations which add something to the recognition, so as to enable the child more clearly to distinguish the cup and more quickly and surely to grasp it and carry it to his mouth, are built into the perception. Others are largely neglected. While it is true that the child does exhibit some interest in sensations, in discriminating between them and in giving them names, he gives attention to them and discriminates between them chiefly because they stand for an object which has some practical meaning for him.

Illustration: Double images are recognized as a

difference in distance. The fact that we pay attention to those aspects of objects which have a meaning or significance for us, and to other aspects only to this end, is shown by the fact that many differences between our sensations are not recognized except in the form of a meaning which is represented by these differences. For example, if we hold two fingers before our eyes, one about twelve inches farther away than the other, and look at the nearer finger, we can make out a double image of the one farther away. This is due to the fact that the image of the far finger is not focused upon the corresponding parts of the retinæ of the two eyes. This, however, is not usually noticed by us unless our attention is called to it; and as a matter of fact many people find difficulty in distinguishing two images even when the fact of their existence is pointed out. We do not pay attention to the double images for themselves; the doubleness is only represented by a meaning — the meaning of nearer or farther away.

Another illustration: A difference in the loudness of a sound as heard in the two ears means direction. A still more striking example may be taken from the experiences which we have from the two ears. If a sound is given midway between the two ears, we receive substantially the same sensation in both, and it is very difficult to give the exact location to the sound. If, however, the source of the sound is at one side, the loudness and the quality of the sensation in

the two ears is different, and we interpret this difference as meaning that the sound comes from a given direction. We cannot tell from the examination of our sensations themselves that we get any different sensations in the two ears. The fact that the perception of direction is based on the difference in the sensation from the two ears may be very strikingly confirmed by giving two sounds simultaneously to the two ears at different distances. This may be done by means of two telephone receivers which are in the same circuit. Under these circumstances we recognize a single sound, not in the real direction of either of the two sounds, but rather in an intermediate direction, which corresponds to the position of a sound which would give the same relative loudness to the two ears as is given by the two separate sounds.

One may distinguish finely in some fields and coarsely in others. A good example of the place which meaning has in directing our attention to a discrimination between sensations is given by Kirkpatrick. He shows that the same person may discriminate finely in certain fields, and much less finely in others. The primitive man, for example, is able to discriminate very finely between signs which represent to him the passage of an animal or of a human being through the woods. The same signs pass entirely unnoticed by the civilized person. On the other hand, the savage is unable to discriminate between the small and intricate marks on a page which to the civilized person who has

learned to read are entirely clear and easily distinguished. The difference is, of course, that the printing has significance to the civilized person, and he has therefore learned to discriminate in that field. This whole matter will have a practical application in the discussion of the question of sense training.

Perception is influenced more by the development of meanings than by increase of ability in discrimination. The foregoing discussion will give a basis for the distinction between the type of perception of the child and of the older person. The development of perception which takes place with the advancing age of the child does not consist primarily in the ability to make fine discriminations. The young child can be trained to discriminate more accurately than does the average adult. The ability which the child develops with his education and with advancing age is the ability to pick out of a whole situation certain aspects of it which will have significance. If a child and an adult walk down the street together and are questioned afterwards about what they have seen, it will be found that the child has seen more miscellaneous things than the adult. His mind is open to impressions, but it is not directed to gathering impressions of any particular sort. If his attention happens to be attracted in a particular direction he will observe what is going on very minutely. The adult, on the other hand, is likely to observe some particular sort of facts. If he is a botanist and is walking through the woods,

he will observe the forms of plants, of flowers, etc., with great accuracy and minuteness; not because he has greater power of discrimination among sensations, but because he knows what to look for, and knows the significance of what he sees.

The child must develop habits of observation. We may say, then, that the development of the child consists in the formation of habits of recognition, or of habits of observation in particular directions. This may, it is true, close his mind to the observation of other sorts of facts, so that the young child may observe a great many things which will be hidden from the older person. The young child may be less easily misled by the method of the sleight-of-hand performer, which consists in attracting the attention of the audience toward one point while the trick is being done somewhere else. One cannot rely on the child to keep his attention on the spot where the performer intends it to be directed, because he has not yet formed sufficiently stable habits of observation.

The child is suggestible in reporting what he has observed. The child is more easily deceived or more suggestible than the adult in other ways. Having had less experience as to what is likely to happen, he is more ready to believe that anything which appears to be true has actually happened. Furthermore, in recalling what he has observed, he is more apt to confuse what he has actually observed with what he has not, but in spite of this unreliability he has a most implicit

faith in his ability to report faithfully what he thinks he has observed. Finally, he is more subject to personal influence than the adult in concluding that he has or has not observed any facts which may be in question. This makes it necessary to use a great deal of caution in questioning the child. It is, of course, a commonplace that the court does not give full recognition to testimony given by children, because of the ease with which they are led to think they have experienced that which they have not. The same caution should be observed in questioning the child in school. There is danger that the child will merely follow the cue given by the teacher in the question, instead of giving expression to his own knowledge or opinion. Unless one is trying deliberately to develop a sentiment by means of suggestion, leading questions — or questions that imply their own answer — should be avoided.

The distinction between subjective and objective observers. We sometimes hear it said that some persons are good observers in general, while others are bad observers. We mean by this that some persons are able to report faithfully what they have actually seen, whereas others are likely to read into what they have seen the results of their imagination. Some experiments have been made in order to determine whether this difference actually exists, and as a result the distinction has been made between subjective and objective observers. Objective observers are those

who see, perhaps, but a small number of objects, but see them very clearly; while subjective observers may take in a much larger range and give it an interpretation, but the meaning which is read into the facts is apt to be erroneous and may cause the observer to think he has seen what he actually has not.

Most persons belong to a mixed type. Something corresponding to such differences undoubtedly exist, although most persons do not belong to either extreme type. When persons are tested with reference to their observation in a number of different fields, it appears that a person may be an objective observer in some fields and a subjective observer in others. We may say, then, that most persons belong to an intermediate type. There are, undoubtedly, however, a few who are extremely unreliable and who read into what they see their own interpretations; while there are others who are very reliable and who distinguish clearly between what they see and the meaning which they give to it.

The value of sense training

Some educators advocate much sense training. As distinguished from training in observation or in perception in the true sense of the term, there are a number of educators who hold that it is desirable to train the child in the ability to discriminate finely between different sensations. This does not involve the recognition of the meaning or the interpretation of the sen-

sation, but merely sensory discrimination. Illustrations of the demand for this type of training may be found in Halleck's *Education of the Central Nervous System*. In chapters 7 and 8 of this book the author advocates the training, not only of the senses of sight and hearing, but also of touch, taste, and smell. He gives a long and elaborate series of tests which may be made in order to develop the child's ability to discriminate and to recognize a great variety of smells and tastes, and of objects which give different sorts of touch experience. Great prominence has been given to the demand for such sense training also by the Montessori method. In this method, as applied to kindergarten children, many exercises are given to enable the child to discriminate between fine shades of color, between different textures of cloth, or of other objects, and between sounds. The belief underlying the advocacy of this method is that when the child's senses are trained, he will be enabled to observe better and to use the results of his observations in his thinking.

Experience with a wide range of concrete objects is of great value as compared with sense training. In considering the value of such suggestions we must keep clearly in mind distinctions between sensory discrimination and the familiarity with objects so that they can be recognized and their meaning understood. The child may have his senses very keenly developed and yet know nothing of the common objects of the world about him. It is conceivable that the child

might be put into a room with a few pieces of apparatus and have his senses very keenly developed, and yet not know such objects as a tree, a bird, a cow, a calf, a house, a trolley car, etc. There is sometimes failure to distinguish between sense training, strictly speaking, and this acquaintance with a large number of objects. This knowledge of objects was studied by G. Stanley Hall and reported in an article entitled "The Contents of the Children's Minds on entering School."[1] It was found that children were remarkably ignorant of many of the objects which were talked about or read about in the earlier grades. This lack is undoubtedly a great handicap to the child, and anything that is said in the following paragraphs in disparagement of sense training must not be taken to mean a belittlement of the value of a wide acquaintance with concrete objects on the part of the child.

Keen senses do not necessarily go with high intelligence. Some light may be thrown upon the value of mere keenness of the senses by facts which are either common matters of observation, or have been established through scientific experiments. It is well known that certain of the lower animals have some of their senses more keenly developed than the human being. The dog has a much keener sense of smell than has man. Some of the birds have a keener sense of vision, and some insects can hear sounds which to us are absolutely non-existent. The keenness of senses in

[1] Reprinted in *Some Aspects of Child Life and Education.*

these animals does not produce in them a corresponding degree of intellectual development. Furthermore, if we compare the sensory keenness of backward children, or even of feeble-minded children, with that of normal children, we find that there is no great difference. In order to find striking differences between children of different degrees of intelligence, we must go to such higher mental processes as reasoning, the recognition of logical relationships, and memory.

Discrimination should be keen enough to meet the demands of perception. The discussion of perception and its development has shown that, in the ordinary course of life, we develop sensory discrimination as it is found to be useful to us in meeting the demands of our practical life; that is, we distinguish between sensations with sufficient accuracy to enable us to get an accurate perception. It would be possible for us to develop our senses very much more highly than we ordinarily do. The blind person has developed a sense of touch in his fingers very much more than the person who has sight. He also has developed the sense of hearing more highly than has the average person. It is undoubtedly true that all of us could develop our senses more keenly than we have, but it is questionable whether we should find that such development repaid us for the effort which was expended.

Sensory defects should be discovered and, if possible, corrected. Whenever the child is deficient in sensory capacity, there is, of course, danger that his

mental development will suffer because of the lack of the proper materials with which to think. We assume, in the ordinary routine of school life, that the child can hear the directions which are given him in the schoolroom, and that he can see figures and words which are written on the board. If the child is sufficiently deficient so that he cannot hear or see these things, he will unquestionably suffer. It is necessary to find out whether a child has a degree of sensory keenness which will enable him to profit in these ordinary ways by the things which go on about him. It is also necessary to correct defects wherever they can be corrected.

The normal child does not need systematic sense training as does the feeble-minded child. We must clearly recognize that certain senses are very much more important than others. Sight and hearing are of importance out of all comparison to taste and smell, and touch is intermediate between these in value. We must also recognize that, although sense training is of considerable value to the feeble-minded child, the normal child gets out of his ordinary experience training which must be given to the abnormal child by special exercises. The normal child learns to distinguish colors and shapes through the exercise of his natural curiosity, which prompts him to play with things and learn their names as he hears them spoken by older persons. A little special attention by the parent will serve to hasten this development and make

the child's knowledge more exact and comprehensive. The feeble-minded child, on the other hand, does not learn the common facts of his environment spontaneously, but needs special, systematic drill. It is a great mistake to assume, as does the Montessori method, that this training, which is necessary for the defective child, is suited to be the main part of the education of the normal child.

REFERENCES

1. E. Barnes: "Study of Children's Drawings," *Ped. Sem.* (1892), vol. 2, pp. 455–63.
2. A. B. Clark: *Children's Attitude toward Perspective Problems.* Barnes's *Studies in Education* (1896–97), pp. 283–94.
3. G. Stanley Hall: *Some Aspects of Child Life and Education.*
4. R. P. Halleck: *Education of the Central Nervous System.* Macmillan (1896), chaps. VII and VIII.
5. C. H. Judd: *Genetic Psychology for Teachers.* D. Appleton & Co. (1903), chaps. I and II.
6. C. H. Judd and D. J. Cowling: "Perceptual Learning." *Psychological Review, Monograph Supplements*, vol. 8, *Yale Studies* (N.S.), vol. 1, no. 2, pp. 349–69.
7. H. T. Lukens: "A Study of Children's Drawings in the Early Years," *Ped. Sem.* (1896–97), vol. 4, pp. 79–110.
8. W. Sargent: *Fine and Industrial Arts in the Elementary School.* Ginn & Co. (1912), chap. I.

CHAPTER IV

READING: PERCEPTUAL LEARNING

The relation of reading, writing, and spoken language

Reading and writing are very closely related to one another from the fact that both are related to spoken language. Writing is the expression by graphic signs of ideas which were previously expressed in spoken words. Reading is the ability to recognize words which have been so written or printed. In both cases, then, we are concerned not merely with the recognition or production of certain forms, certain marks on paper, but we are concerned with the recognition or the expression of ideas as they are represented by printed or written forms. The written or the printed forms on the paper stand for spoken words, or may be said to be the symbols of the spoken word.

Reading and writing should have meaning to the child. The teaching of reading and of writing ought, therefore, to be very closely related to the meanings which are expressed in spoken language. The child ought, when he writes, to be able to understand the meaning of what he is writing. Or, more correctly, he ought, in the main, in his writing, to be expressing ideas. Similarly, we now insist that when the child learns to read he shall be gaining the ability not merely

to pronounce words, but to read a meaning into the words which he is pronouncing.

Drill is necessary for the perfection of skill. This principle, that learning to write and learning to read should be related to each other and that both should be related to the spoken language of which they are the expression, should not be taken to mean that no drills should be given for the perfection of the ability to write or to recognize words. We have seen, in the case of writing, that it is often necessary to stop in the process of expression of thought and to take time to perfect the technique of form production or of the development of the proper movement. What is here meant is that writing and reading should always have a meaning for the child, but it is not intended to imply that this meaning should always be uppermost in his mind. There should, in fact, be times when the technique of the production or of the understanding of words is the thing which is uppermost. If we attempt to develop skill merely as an incident to the recognition of meaning, we find that there are large gaps in the child's ability. There are two extremes, then, which have to be avoided. On the one hand, the development of great technical ability which has no meaning or significance for the child and which therefore cannot be used by him to serve its real purpose; and, on the other hand, the failure to develop the technique to such an extent that the acquisition may be used as an efficient tool by the child.

Writing enhances the meaning of reading. A few remarks may be made here as to the manner in which reading and writing are related to each other. In the development of writing in the race, reading and writing, of course, went together: that is, the race was able to recognize those signs which it could produce; and it could recognize only those which were produced because it was dependent on writing for the creation of material to be read. The child, however, when he comes to read, is confronted by a large amount of material which has been developed in the past history of the race, the origin of which he is entirely unacquainted with. A problem is then raised that may be put thus: Shall we introduce the child to this accumulated mass of printed matter as fast as he can learn to read it, or shall we repeat the history of the race and make writing keep pace with reading? It is not necessary to make the faster process of learning to recognize words keep pace with the slower process of learning to write them. When the child does learn to write, however, it undoubtedly not only makes his perception of the letters more distinct, as we saw in the chapter on writing, but also gives him a better realization of the fact that in reading one receives a message from another person in the same way as in listening to another person talk. This fact furnishes an argument against deferring writing any longer than necessary. The connection between reading and writing may be emphasized by relating both of them to oral expression.

Writing and reading should be related to oral expression. Both reading and writing may be related to oral expression in the following manner. The child or the class may be led to describe some experience they have had; this description may then be written out by the teacher and reproduced by the children, or written out by the children with the help of the teacher; and the children may then read what they themselves have composed, first orally, and later in writing. When the connection between oral composition, writing, and reading is thus made, the child comes to think of writing as having some use, as being a real activity, and related to the expression of his ideas instead of being merely a formal production of certain meaningless signs upon the paper. Similarly, reading comes to be the acquiring of ideas, or the gaining of experiences from others rather than the ability to pronounce meaningless words. When reading and writing thus have acquired their normal meaning through their association with each other, the time is ripe for the development of a higher degree of technical skill in order that the ability may be gained to express and to recognize still more difficult ideas or ideas expressed in a more difficult form.

The stages in learning to read

Learning what printed words are for. Following this general account of the meaning and significance of reading and writing, we may describe the stages

through which the child's recognition of words progresses until it becomes complete in reading. In the first place, the child learns that printed words represent spoken words. This knowledge may be gained before he is able to recognize any particular printed word as meaning any particular spoken word. The experience of seeing other persons read, the realization of the meaning of street signs, of store signs, etc., will give him this early type of recognition. The child comes to understand this without any special form of training, merely by casual observation of the actions of people about him.

The word-learning stage. The child goes beyond this general recognition when he learns to connect certain words with the printed forms which represent them. He may do this before he is able to read connectedly. He may, for example, recognize his own name when it is printed, or the names of the members of his family, or of familiar objects. This recognition gives him a starting-point for more coherent and connected reading. This association between the spoken name and the written word is really not much more complex or difficult than the connection of a spoken name with the object. It, of course, soon becomes very much more complex in virtue of the fact that different printed words are made up of combinations of the same letters, but at the start the child can associate a printed form with a name in very much the same way in which he associates the object with its name. The difficulty

arises when the child begins to learn a large number of words. The amount of difference between different words is in many cases small, and it is necessary to pay a good deal of attention to the details of a word in order that it may be distinguished from one that is similar to it.

The letters need not be learned first. This essential similarity between the association of a printed word with a spoken word, and the association of a name with an object, is taken advantage of in present-day methods of teaching the child to learn to read. In the older methods it was assumed that the child had to be brought to this recognition by an indirect route; in other words, it was believed that it was necessary for the child first to learn the elements of which the word is composed, and then to be able to put them together into the word. The association was not between spoken word and printed word, but first between spoken names of letters and printed letters, then between the printed letters as they were combined into the word, and finally, by this indirect route, between spoken and printed words.

There are special difficulties in beginning with the names of letters or the sounds of letters. The matter was made still more complex by the fact that the letters which composed the words did not represent the sounds of the spoken language, but rather represented certain names which did not in many cases correspond with the sound. For example, in building up the word

cat, the names of the letters *c-a-t* do not stand for the sounds of the spoken word, but are rather names which are more or less unrelated to the sound. This difficulty was somewhat remedied when the sounds of letters were used instead of their names. The sounds of the successive letters represent in some measure the sound of the word as a whole, and the child can see that the combination of the letters as represented by their sounds represent the spoken word. Even here, however, many difficulties arise in such a language as the English in which the spelling is not phonetic. In such a word as *though*, the combination of the sounds of the separate letters does not at all represent the sound of the spoken word. At the best, even if the language were entirely phonetic, there is no reason why it would be necessary to begin with the elements of which the words are composed. As has already been said, the child may form the direct associations between the spoken and the printed word as the starting-point of his reading.

The sentence is too complex to begin with. Some have attempted to go still further than this, and make the association first between a written sentence and the spoken sentence. This method seeks justification in the fact that an idea is commonly expressed by a combination of several words in a sentence rather than by a single word. In spite of this fact, however, the word is a fairly distinct unit of speech, and if we trace the development of speech in the child we find that

he first isolates single words from the trains of sentences which he hears spoken by those about him. We found in our description of the language development of the child that he first uses sentence words. Single words represent for him trains of thought; and the supplementary parts of the idea are expressed through facial expression, gesture, etc. In learning to speak it is single words that first catch the child's attention, and the same is true in reading. The child may not be able to hold in his mind, at the beginning, the minor parts of a sentence and put them together into a sentence as a whole. We may, therefore, conclude that his reading, as his speaking, should begin with words which form the core of a sentence or an expression of thought. After he has gained the ability to recognize a few important words, the others may then be introduced, and he will come gradually to the recognition of their meaning also.

After the early stage letters and their sounds must be recognized. This gives the child the starting-point for the recognition of words. He will not progress far, however, until he understands that words which are printed represent spoken words, not because they are associated together in an arbitrary way, but because the printed word represents a combination of sounds in the same way that a spoken word represents a combination of sounds. In other words, he must come sooner or later, and in fact rather soon, to the understanding of the principle of the alphabet. Although it

is not necessary for him to start with the recognition of the alphabet, yet it is necessary that he gain this recognition before he progresses far.

Written symbols were originally pictures of objects. We may gain a clear idea of the meaning of the alphabet by a survey of the course of its evolution. At the beginning the written signs which were used to express ideas resembled the objects which they were supposed to represent. This is seen in picture-writing. Picture-writing, as the name indicates, conveys its message through a series of pictures of objects which when put together suggest a connected story. Sometimes other signs, such as a rough map, are used to indicate the relations of the pictures to one another. The Chinese language still retains this same principle of picture-writing in that the signs which represent the words of the language were originally pictures of the objects. For example, the sun was represented by a circle with

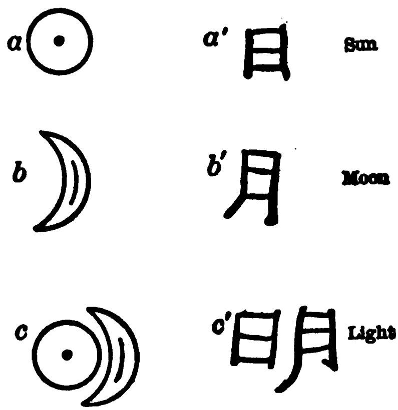


FIG. 7. ILLUSTRATION OF PICTURE-WRITING

(After E. B. Huey, *Psychology and Pedagogy of Reading*, with the permission of the publishers.)

a dot in the middle. (See Figure 7.) As it became easier to use straight lines rather than curved lines, another sign was substituted for this, but the meaning was still based on the original correspondence in appearance of the sign and the object. The Chinese language, then, instead of having twenty-six signs, as has our language, has thousands of signs representing individual objects or ideas, together with certain key-signs which may be used in combination with them.

These pictures were then used to represent other objects having names of the same sound. The written language which we inherit, however, developed according to an entirely different principle. The development toward the new forms of printed signs probably had its origin in the necessity for writing proper names. A proper name cannot be expressed by the picture of the object which it represents, because the distinguishing marks of an individual are difficult to represent, and because it may be applied to a number of different persons. This led to the device of breaking up the name into a number of parts each of which could be represented by some object. The name then was represented by a combination of the signs which

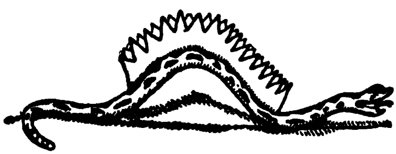


FIG. 8. ILLUSTRATION OF PICTURE-WRITING

(After E. B. Huey, *Psychology and Pedagogy of Reading*, with the permission of the publishers.)

represented other objects. See, for example, the sign which was used to represent the Aztec name, Itz-coatl. The first part of this name is the word for *knife*, and the second part, the word for *snake*. The name was

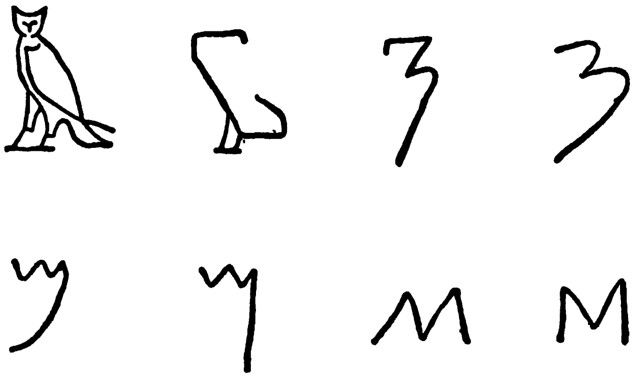


FIG. 9. EVOLUTION OF *M*

(From Judd's *Genetic Psychology for Teachers* [Appleton, 1903], p. 208.) "The figure shows the derivation of the letter *M* from the Egyptian hieroglyphic owl. The four forms in the upper part of the figure are Egyptian forms. The first form on the left of the lower series is an ancient Semitic form. Then follow in order an ancient Greek form, and two later Greek forms." (I. Taylor, in *The Alphabet*, pp. 9 and 10.)

then represented, as shown in Figure 8, by a combination of the picture of the knife and of a snake.

The simplified pictures were finally used to represent the first sound of a name. In early Egyptian and Phœnician times this process was carried a step farther by using the sign for a word, not to represent the word as a whole, but the first sound in the word. In this way there came to be developed a great many signs which stood for single sounds. Thus the letter *M* was devel-

oped from the sign for the Egyptian word *mulak* (owl) through a series of steps illustrated in Figure 9. In many cases the same sound was represented by a variety of signs, because different words starting with the same sound might have signs to represent their initial sound. The Egyptians apparently did not develop a series of single signs to represent the various sounds. This was carried forward by the Phœnicians or the Greeks.

Our alphabet does not entirely suit our language. As a result of this development we have a series of signs which stand for the sounds of the language. In the ancient Greek and Phœnician alphabets there was probably a complete correspondence between a series of sounds and of signs. In our own case, however, we have inherited an alphabet and have applied it to a language which is somewhat different from that for which the alphabet was devised. Therefore, we have some sounds which are represented by a number of letters, and some letters which represent a number of sounds. In other words, our language is not phonetic.

The child may make the association between the sound and the letter either by systematic drill or incidentally. To return to our description of the development of the child, we see that it becomes necessary for him sooner or later to awaken to a recognition of the sounds which are represented by the different letters. This may be done in two ways. In the first place, the

child may be told what sound is represented by the letter; or, in the second place, he may be led to make an association between the sound of a letter by seeing the same letter appearing in a large number of words. We may call the first a definitely phonetic method of teaching, and the second an incidental method.

A moderate amount of phonetic drill improves the ability to recognize words. Some amount of phonetic teaching is undoubtedly of value in helping the child both to analyze the spoken words into their sounds and to make the associations between the sound and the letter which it represents. This prevents a waste of time and leads to an earlier formation of the association than would occur if the association were made wholly by the incidental method. The need of making the association between the sound and the letter becomes prominent when the child meets new words, but it is possible to develop such associations incidentally, as is shown by the fact that the child who has had no phonetic drill may be able to recognize new words. But the phonetic drill may be carried to an extreme, and when this is done the child becomes expert in the recognition of the sound of words beyond the development in ability to recognize their meaning. There is some danger of overemphasizing the sounds of letters in contrast to the meaning of the words which they represent.

An added difficulty is due to the fact of a lack of uniform correspondence between sounds and letters.

In the English language, which is highly unphonetic in its spelling, the difficulty arises which has already been mentioned. A letter may represent one sound in one word and another sound in another word; and to teach that a letter represents a particular sound does not always enable the child to give it its appropriate sound in all words. Attempts have been made to overcome this difficulty by using additional signs to indicate when a letter is to be given one sound or another sound. These signs are called "diacritical marks." Other devices which have been used consist in the formation of a new alphabet, or in the invention of additional letters to those of our alphabet in order to supply this want. Whether or not diacritical marks or additional signs in the alphabet are used, we must recognize that they are for temporary rather than for permanent use. The child must ultimately come to recognize words which are printed in the ordinary letters in common use without diacritical marks. The question, then, is, whether he can better learn to recognize the various sounds of ordinary printed letters as they appear in different words by first using diacritical marks, or whether he can better learn by the incidental method.

Diacritical marks should be used sparingly. It is undoubtedly true that we sometimes create difficulties for the child by calling them to his attention, and by the use of devices which have for their purpose the overcoming of these difficulties. The child may recog-

nize the difference in the sound of *a* in *say* and in *saw*, for example, without its presenting any particular difficulty to him. But if a diacritical mark is used in the one case, or in the other, or in both cases, he has an additional thing to learn. Undoubtedly some confusion in the mind of the child is inevitable, and the use of a few diacritical marks is desirable to overcome this confusion. A few years ago there was an extreme use of such signs and of new or artificial alphabets, but teachers are coming to discover that this extreme use of supplementary signs is unnecessary and serves to confuse rather than to help the child. Their chief use is to enable the child to gain the pronunciation of a word which is new to him in his reading and in his speech, by consulting the dictionary. This need does not arise in the early years.

Spelling necessitates a clear recognition of the letters. The necessity for the recognition of the letters as the elements of a word is due also to the requirements of writing. When a person writes words he must know not merely the general form of the word or the more prominent letters which compose it, but he must be able also to give each individual letter in its proper order. The requirement of spelling necessitates more minute analysis of the word than the requirement of its recognition. Furthermore, it has been shown by the photographic method that the child makes many more eye movements even in the recognition of words than does the adult. The child in the

first two or three grades moves the eye almost as many times as there are letters in the word; whereas the adult, as we shall see later, makes only four or five pauses in an ordinary line of print. This is an indication that the child does not recognize words as wholes to the same degree as does the adult, even though he has been taught by the word method. On the score, then, of the recognition of words and of the ability to spell words in writing, it is necessary that the child should finally get a clear recognition of the letters themselves, and not merely of the words as wholes.

To read sentences fluently requires the development of eye-movement habits. The ability to read is, in addition to the ability to recognize words, the ability to recognize them rapidly and connectedly as they appear in their order in the sentence. The sentence consists in a combination of words which express a coherent meaning. When the adult reads he takes in during each of a series of eye pauses a group of letters which make up a word, several words, or even phrases, at the same time. The mechanism by which this recognition of successive words and phrases take place has been recently studied. The opinion was previously held that the eye travels steadily along the line and that the words are recognized while the eye is moving. Experiments which have recorded accurately the movements of the eye have shown that what really takes place is a series of rapid movements separated by eye pauses, and that the recognition of the word

takes place during the eye pauses. The number and length of these pauses vary with different persons and in the same person with different kinds of subject-matter or with different sizes of type and length of lines, etc., but it is fairly constant for the same person under the same conditions. Each person has developed a certain habit of eye movement which accords with his habits of reading, or which expresses the habits of recognition which he has formed. It is a question whether the eye movement is the factor which determines the kind of a person's recognition, or whether the eye movement is governed by the recognition of meaning; but at any rate it is clear that one of the necessities which confronts the child as he learns to read is the necessity of building up these eye-movement habits.

Progress consists in a decrease in the number of movements and an increase in the scope of recognition at each pause. As was mentioned previously, it has been discovered that with the very young child these habits of recognition of words, or large parts of words at a time, has not yet been completely formed. The child makes progress in the development of the habit from the stage in which he makes a large number of movements, corresponding to the individual letters or small groups of letters, to the stage in which he makes fewer movements corresponding to larger groups of letters or words. Recent experiments indicate that probably by the third grade, many if not most of the

children have developed these movement habits sufficiently to read with a high degree of rapidity.

Words are connected into sentences by inner speech. These facts in regard to eye movements throw light upon what happens in order that we may recognize words in reading. Evidently what we get of the printed page is a series of glimpses. We are not aware of these pauses in our reading, and in fact we are not aware that we perceive the sentences of a page piecemeal, but it seems rather as though we read continuously. The reason for this is that as we perceive the words we put them together and weave them into a whole as though we were speaking the sentences. Words are connected in our minds in the first place through speaking them in sentences and through listening to others speak. When we read there is a tendency also to repeat the sentence. With unpracticed readers this is carried to such an extent that they move their lips in the pronunciation of the words that are being read. Even though a practiced reader does not give these outward signs of pronunciation, yet it has been shown by experiment that the vocal cords and the tongue make very slight movements which correspond to the words which are being read. The glimpses which are got of the line in reading, then, serve to set up the train of activities which correspond to the speaking of the sentence.

Inflection, pitch, and tone quality correspond to the meaning of the sentence as a whole. Not only are the

words reproduced in some form of inner pronunciation accompanied by the imagination of the sound of the words or of the feeling which is produced in pronouncing them, but we also have imagery which corresponds to the relationships of the words in the sentence. When we speak a sentence the words are not pronounced in a monotone, but they are given a certain emphasis and inflection which is due to their relationship to the other words, independent of their character as words taken separately. If we listen to a person talking in another room, although we cannot distinguish the words, we have some notion of the type of sentence which he is uttering. We can distinguish between a question, a declaration, and an exclamation, etc. In a similar way the pitch and quality of the voice represent the emotion which accompanies the words as they are being articulated. Although one may, when he has acquired the habit of rapid silent reading, slur over the articulation of the individual words, yet this modulation of the voice remains, and is represented in the imagination or in actual changes in the vocal cords or in the other organs of speech which correspond to such modulation. These tendencies to inner speech are somewhat different from those which complete themselves when one is actually speaking, as is shown by the fact that they may take place during inspiration of breath as well as during expiration; but though they are thus modified in silent reading, they have an important connection with the apprehension of meaning.

Inner articulation is more important in some kinds of reading than in others. The importance of inner articulation varies with the rapidity of the reading; or with reference to the aim, according as it is merely the rapid apprehension of meaning or some other form of appreciation of what is being read. When one is reading passages the value of which depends upon the sound quality, it becomes necessary to read in such a way that this sound quality will be recognized. In poetry, for example, where the rhythm and the melody and the alliteration, or other sound characteristics, are important in the appreciation of the æsthetic qualities of the poem, it is necessary to read in such a way that one has time to get a clear experience of these characteristics in the imagination. The same thing may be true to some extent of literary prose. It is, however, not correct to apply the principles which govern such forms of reading to reading of all sorts. It is undesirable in much of our reading, which is so voluminous that the meaning must be apprehended as rapidly as possible, to dwell upon these elements of the sentence. It is not necessary in a great many cases to pay particular attention to every detail which is being read, but it is desirable to gather only the main ideas.

Oral and silent reading

Ability in silent reading is much more important than in oral reading. This raises the whole question of oral and silent reading, which must be considered

before we finally arrive at a conclusion with reference to rapid reading. Traditionally the reading in the schools has been chiefly oral reading, and little or no attention has been paid to silent reading, or to reading for the purpose primarily of getting the thought, and not for the purpose of expressing it to others. The relative value of these two sorts of reading must be considered in reference to the use to which they will be put in the child's after life. From this point of view it is clear that silent reading is by far the more important. Oral reading is the merest incident in the life of the average adult; whereas silent reading plays a very important part in his life. The only justification for a comparatively large amount of emphasis upon oral reading would be the assumption that it is a necessary condition for the proper development of oral reading.

Proficiency in silent reading does not correspond closely to proficiency in oral reading. One may hold that while oral reading is not to be used, yet it is necessary to the development of silent reading. If we compare the oral and silent reading of the adult, however, we are led to believe that there is quite a wide difference between the two processes. If the adult who has become practiced in silent reading is required to read orally, he will see that the apprehension of meaning is less perfect than when he does not pronounce the words aloud. The oral reading, then, is to some extent an interference with the apprehension of meaning. Furthermore, it is evident upon the briefest observa-

tion that a child may read orally and pronounce the words with a fair degree of accuracy of inflection, etc., and yet have very little comprehension of what he is reading.

Oral reading is of little value for silent reading after the early stages of learning. We must assume, therefore, that, although there is inner pronunciation in silent reading, yet the development of silent reading is mainly to be got by the suppression to a large extent of the oral accompaniment, and the fixing of attention on what is being read. Experiments in which the child is encouraged to get the meaning from the printed page rapidly and without pronunciation, or at least without oral reading, indicate that he is capable of a much higher degree of development in this direction than we have been accustomed to think. Oral reading is doubtless the natural and best kind of reading for the beginner and perhaps during the first year. It furnishes the most natural motive for the child to get the meaning of the printed words, and his rate of recognition is still so low that it cannot be retarded by speaking. Aside from this early reading, however, oral reading is chiefly of value, not because of its bearing on the greater part of our silent reading, but because it is of itself of importance in the development of oral expression, and because it leads to an appreciation of the æsthetic qualities of certain types of material.

Efficiency in reading

Efficiency is measured by speed and apprehension of meaning. We have assumed that the chief purpose of reading in the schools is to develop efficiency in silent reading. This assumption is entirely justified by the use to which reading is put in after life. If this is true the question next arises: In what does efficiency consist? Undoubtedly one element in efficiency is speed, and this has been incidentally referred to. Rapid reading, however, is of little importance unless the meaning is apprehended with clearness and accuracy. We must include, then, the ability to reproduce or to use what has been read, as well as the amount which has been read, in our measure of efficiency. The speed is easily measured by measuring the amount that a person can read in a given time. The degree to which the meaning is apprehended may be measured in several ways. The pupil may be required to give an account of what he has read; or he may be asked to answer questions on the material which he has read. These are the tests which are commonly used, and when they have been applied, it has been found that the rapid readers apprehend the meaning about as well as slow readers. Furthermore, when a person through a course of drill increases his speed in reading, he is as likely to increase the accuracy with which he apprehends the meaning as the reverse. In fact, the increased speed of reading often

seems to heighten the attention or to increase the amount of mental energy which is expended, and this is expressed in the better apprehension of meaning as well as in greater speed. The figures in Table I show that the speed of reading in the lower grades may be greatly increased, and that this may be accompanied

TABLE I

Improvement in reading from September to March[1]

	Rate in words per minute	*Amount reproduced*	*Percentage of correct answers to questions*
Grade 3			
September	76.4	71.1	44.6
March	149.1	135.	44.
Per cent gain	72.7	63.9	.6
Grade 4			
September	92.7	133.8	56.7
March	163.3	212.9	60.9
Per cent gain	70.2	79.1	6.6
Grade 5			
September	113.	52.2	16.3
March	129.2	70.5	25.6
Per cent gain	16.2	18.3	9.8
Grade 6			
September	128.	52.1	27.1
March	130.1	85.3	35.
Per cent gain	2.1	33.7	8.
Grade 7			
September	122.7	75.6	42.7
March	142.8	125.5	48.3
Per cent gain	21.8	49.9	5.6
Grade 8			
September	147.2	116.5	55.3
March	158.9	179.6	62.5
Per cent gain	11.7	63.1	7.1

[1] From an unpublished master's thesis by K. D. Waldo, on file in the library of the University of Chicago.

by an increase in the ability to apprehend the meaning.

Rate of reading is not so important in the comprehension of difficult matter as in easy reading. There is another element in efficiency which is not accurately measured by the means which have been mentioned, namely, the ability to relate what is read to one's past thinking, or to examine it critically in order that one may judge of the correctness or worth of what is read. In some kinds of reading, also, it is necessary to spend considerable time in order that the thought may be apprehended. The tests which have been used have referred chiefly to the types of reading-matter which can be readily apprehended without stopping to think through the ideas which have been expressed. It is very likely that some modification will have to be made in the conclusion with reference to rapid reading when the other sort of test is made, and when different kinds of reading-matter are studied. For example, in solving problems in arithmetic or in other forms of mathematics, much of the accuracy, or much of the efficiency, depends upon the ability to grasp the meaning of the problem as expressed in its statement. This is one form of reading, and it must be included in any comprehensive view of what efficiency in reading means.

The rate of ordinary reading is usually too slow. Even with this qualification, however, it must be said that there is ample evidence that the majority of per-

sons read very much more slowly than is necessary. The reading rate has probably been fixed largely by the speed in oral reading, and this is one reason why it is undesirable to overemphasize oral reading in the schools. A little effort on the part of adults is sufficient very materially to increase the rate of reading and the experiments which have been made indicate that children are also susceptible to this type of training.

Speed must be flexible. As was hinted in a previous paragraph, the child must be taught to read each particular kind of subject-matter with its appropriate speed. In other words, he must read only as rapidly as he can grasp the thought. The test which was made in one school showed that the sixth grade and the seventh grade read the same passage with about the same degree of speed, but the sixth grade was able to apprehend only a very small proportion of the ideas of the passage. These sixth-grade pupils had not had the proper training, or at least did not have the proper attitude in this particular test. They should have realized that they were not getting the meaning, and they should have adjusted their speed of reading accordingly. With some kinds of subject-matter it is necessary that one learn to skim through and gain merely a general notion of the thought which is expressed. The newspapers, for example, furnish a great deal of material which should be read in this manner. When one has only learned to read carefully word by word, he either has not the ability to go over

the amount of reading-matter which may advantageously be read by the average educated adult at the present time; or if he does, he expends very much more time than should be spent.

Rate of reading is increased by attending to the meaning as distinguished from the mechanics. It is not known with certainty what change takes place in a person's reading habits when he increases his speed of reading. Some change in eye movement must, of course, take place. Either the movements are less frequent, that is, fewer pauses are made to the line, or the duration of each pause is less, or both of these factors change together. If we compare rapid and slow readers, we find that some rapid readers owe their ability to the fact that they make their pauses of short duration, and others to the fact that they make few pauses to the line. This, therefore, does not give an answer to our question. Dearborn found that a reader tends to fall into a set habit of eye movements in reading a certain passage of a particular length of line, size of type, etc., and concluded that the habit was a more or less accidental thing and that the speed was determined to a large extent by the eye-movement habit. On the other hand, the experience of those who increase the rate of their reading indicates that they do it chiefly by paying attention to the apprehension of meaning. They attempt to fix their attention on the things which are read about, and to pay little attention to the mechanics of the reading itself. The suppression

of lip movement and of the images of the sound of the word or of the pronunciation of the word assists in rapid reading. When a person is thinking of each word and hears it distinctly or feels how it is pronounced, he can only read at a certain limited rate. When the word, on the other hand, suggests certain ideas, or images of things, and the reader frees himself from the images connected with the word itself, he is enabled to read more rapidly.

The variation in speed according to the subject-matter shows the importance of apprehension of meaning. The conclusion that rapidity of meaning depends more upon the ability to apprehend the meaning quickly than upon the habit of eye movement is further confirmed by the fact that the rate of reading varies considerably with the kind of subject-matter. If the same person is tested in easy narrative and in more difficult scientific exposition, it will be found that he reads the former more rapidly than the latter. The eye movements in the two passages would not be affected by the difference in the subject-matter, and therefore we must conclude that they are not the predominant elements in determining speed.

The pupils should be trained in the rapid apprehension of meaning. If this conclusion is correct, the training of the pupil in rapid reading can be brought about best by some means which will call his attention to the meaning rather than to the mechanics of the reading. It is found that merely to urge the pupil to

read rapidly is an effective way of increasing his speed. Any motive which will lead him to desire to get the meaning quickly will be an effective one. This will at the same time avoid the danger of the pupil increasing the rate of his eye movements or of the pronunciation of words without increasing his rate of apprehension, so that the pupil tends to read in a mechanical fashion. There are a number of devices which might be used to assist the pupil in getting the meaning. Catch passages are sometimes used in which some absurdity is hidden, and the pupil is directed to detect the absurdity as rapidly as possible. Another means is to have the pupil give a summary of what he has read, at the same time trying to read rapidly; or to direct him to read a passage for the sake of getting at some point or the answer to some question which is given beforehand.

Improvement with age above the fourth grade is in ability to grasp more difficult subject-matter. The conclusion that the rate of reading is connected more closely with apprehension of meaning than with the mechanics of reading is also indicated by the fact that when pupils of different grades are given material which is suited to their thought, those who are in any grade above the fourth can read with about the same degree of rapidity. That is, by the time the pupil reaches the fourth grade the mechanics of reading are sufficiently developed so that he can read rapidly anything that he can understand. If, however, the lower-

grade pupil is given a passage which is suited in subject-matter to a higher-grade pupil, the rate of his reading as well as the accuracy of his understanding usually drops. In the apprehension of meaning, as well as in the speed of reading, there is a great deal of overlapping among children of successive grades One investigation, for instance, indicated that 42.6 per cent of the children of the fifth grade surpassed the average of the sixth grade in rate of reading. (See Table II.)

TABLE II

Percentage of children in lower grades exceeding the average performance of children in higher grades in the same test[1]

	Rate			*Words reproduced*			*Percentage*		
	6	7	8	6	7	8	6	7	8
Fifth	42.6	36.1	29.5	24.6	8.2	0	26.2	13.1	0
Sixth	—	35.5	29.4	—	15.7	2	—	15.7	4
Seventh..	—	—	27.7	—	—	14.8	—	—	24.1

In reading as in writing the proper relation must be kept between mechanics and meaning. The process of learning to read illustrates very much the same principles of learning as does learning to write. In the first place, the child has to master certain mechanics in order that he may be able to gather meaning from the words on the printed page. In the case of writing, the problem is to master the mechanics of expression

[1] From K. D. Waldo.

so that a meaning which is represented in the mind of the child will be expressed. In both cases the aim is either the expression or the understanding of meaning. When the final development is reached the mechanics will fall out of attention, but for a time it is necessary that the mechanics be perfected in order that they may serve as an efficient medium for the recognition or the expression of meaning. In both cases the mechanics may be too much neglected, or the attention may be retained upon them too long. Efficient teaching of reading, as of writing, consists in paying just enough attention to the mechanics to make of them a tool for the apprehension of meaning, but to give no more time and energy to their development than is necessary to serve this end.

REFERENCES

1. W. F. Dearborn: *Psychology of Reading. Columbia University Contributions to Philosophy and Psychology*, vol. 14, no. 1.
2. E. B. Huey: *The Psychology and Pedagogy of Reading*. Macmillan. 1908.
3. C. H. Judd: *Genetic Psychology for Teachers*. Appleton (1903), chap. VIII.
4. K. D. Waldo: "Tests in Reading in the Sycamore Schools." *Elementary School Journal* (1915), vol. 15, pp. 251–68.
5. F. Jenkins: *Reading in the Primary Grades*. Houghton Mifflin Company. 1913.

CHAPTER V

MUSIC: PERCEPTUAL LEARNING

The chapter deals with the process of learning to read music. We shall consider in this chapter chiefly the ability to read music as shown by the ability to sing a melody from the printed score. We shall assume that the child has learned to sing in a simple fashion before he is to be taught to read. This neglects the earlier phase of learning to recognize and sing tunes from having heard them, but in the main we shall assume that the child has learned to sing simple tunes just as we assumed in our discussion of reading that he had learned to speak. There are several considerations which justify this apparent neglect of the method by which the child learns to sing from ear, in favor of a description of the method by which he learns to read music. He learns to sing by ear by the simple method of imitation, and there is not much of technique in the process. This is shown by the fact that many children can follow a tune before they go to school. Furthermore, the general principles of music which are represented in singing by ear will be brought out in the description of singing by note. Finally, the importance of beginning the reading of music early in the grades is being more and more recognized, so that

now in many places the child learns to read music almost as quickly as he learns to read printed matter. Reading simple melodies is comparable in difficulty to reading printed language, but reading music has been deferred in traditional school practice because of convention and tradition, and not because it was so inherently difficult as to make this postponement necessary.

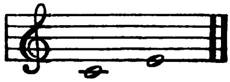
The enjoyment of music is not treated in this chapter, because, although this is one of the chief aims of education in music, it is reached indirectly, and it is the intellectual phase of learning music which is subject to analysis.

The child must first be able to carry a tune. Learning to read music, as has been said, presupposes that a person is able to distinguish different tones, — or in technical language that he can distinguish tones of different pitch, — and that he is able to carry a series of tones in mind to form a melody. The ability to read is, as in the case of reading words, the ability to make the association between a series of printed notes and a series of tones. One cannot connect the notes with the tones unless the tones are already recognized. The child, before he begins to read music, then, must have had some training in the ability to carry a tune. This training may take place in the kindergarten or in the first grade or two, or even before this time. Many children learn to carry simple tunes before they even enter the kindergarten.

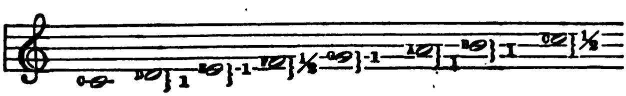
The recognition of intervals and melody

Reading is the recognition of the relationship of tones and not of separate, individual tones. The recognition that the printed score stands for music which is heard or which is sung, as in the case of reading, is one that the child gets merely by seeing music used. After the pupil has learned that the notes on the score represent melodies, the next step is to be able to associate certain particular notes, or rather certain connected series of notes, with a particular melody. The recognition of series or sequences of notes, and the correspondence between these and the sequence of tones in a melody takes place, not through the association of the pitch of the individual notes on the scale with the pitch of the individual notes which are sung, but rather through the recognition of the relationships of different notes to one another. The recognition of the relation of single notes to single tones makes possible the ability to tell absolute pitch, which is a relatively infrequent sort of ability. Very few persons can tell when a note is struck on the piano what note on the scale it is. The ability can be developed by training and is the refinement of the ability which the moderately trained person possesses of telling roughly the pitch of a tone. The method of identifying a tone which most persons pursue is first to have a basic pitch given and then relate other tones to this one.

The recognition of intervals is distinct from the

recognition of individual tones. The pupil learns through practice to associate the interval between two notes on the printed score with an interval in the tones as he sounds them or hears them sounded by somebody else. When the pupil reads the interval C–E  he can sing it, not because he carries in mind the pitch of the two notes separately, but because he carries in mind the difference in pitch which is represented by the combination. The tone which is sounded as C may be any tone whatever and may not correspond to a tone which represents C on any instrument. But whatever tone is chosen for C, the interval between C and E is always recognized as the same.

In learning to read, the child becomes familiar with the various intervals of the scale. When the ability to recognize the intervals between notes is completed, the pupil can reproduce or name all the intervals on the scale. This includes the intervals between adjacent notes as well as the notes which are separated by one or more intervening notes. Let us pause a moment to see what this means. If we consider merely adjacent notes, it means that two of the intervals are recognized as being but half as great as the other five. These two intervals are designated ½ in the accompanying figure



and the rest are designated 1. These smaller intervals are sometimes called half steps and the larger ones whole steps. The wider intervals than those between adjacent notes are also affected by the existence of the half steps. Thus the interval C–E is not the same as the interval D–F. We may summarize these facts in the statement that our music is written in a certain conventional scale,[1] and that the pupil has to learn to read melodies in terms of this scale. This makes learning to read music a more complicated matter than it would be if the intervals between adjacent tones of the scale were all equal. The child, of course, is prepared to recognize the intervals of the scale because of the fact that he has already become familiar with it by ear, and he does not first learn them by calculating that an interval is made up of so many whole steps or half steps. Still, if the scale which the child first learned by ear were made up of equal steps, the recognition of various intervals on the printed scale would undoubtedly be easier.

The child should learn to read simple music before receiving formal instruction in the scale. The instruction of the child might be begun, as it has sometimes been begun in the past, by the attempt to teach the child in a formal manner the intervals of the scale. This, however, is not necessary. The child acquires sufficient familiarity with the intervals of the scale

[1] Only the major scale is considered here for the sake of simplicity. The same principle applies to the minor scales.

to enable him to sing, without knowing the names of the notes, without practice in striking intervals, or even in running scales. The child may go further than this and learn to read simple music and strike the intervals which he meets in it by becoming familiar in an incidental manner with all the possible intervals which are to be found.

Formal instruction should later supplement reading. While it is true that the child should not begin with a formal study of the scale and its intervals and with drill in reproducing intervals which are represented on a printed score, it is equally true that some formal study is desirable as soon as the child has gained sufficient mental maturity and has learned actually to use the scale with some ease. The explanation of the whole steps and half steps and the drill in striking intervals quickly and accurately, which before the child has learned to read at all is a dull and almost fruitless grind, at its proper time clarifies his recognition of the intervals of music. There is no scientific evidence at hand to enable us to say just when this formal instruction should be given, but on the basis of the general principles of the child's mental development it should probably begin at nine or ten years of age.

A difficulty is caused by the use of different keys. We have thus far assumed that musical compositions are always written with the keynote at the same place on the staff. This, of course, is not true. It is often necessary for various reasons — for example, to adapt

a song to the pitch of a voice or to harmonize several instruments — to shift the pitch of the keynote and the whole piece up or down. Each position on the printed staff represents (within slight variations) a fixed and definite pitch. The only way in which a piece can be written in a different key from the one we have assumed as a standard — the key of C — is to place the keynote at a different point on the staff. This would not create so much difficulty if the intervals of the scale were all equal, but since they are not equal, it is necessary, when the keynote of a piece is shifted, to modify the pitch which is represented by some of the lines or spaces. Otherwise the whole and half steps would not come at the right place in the scale. The modifications consist in shifting the pitch of certain notes up or down a half step. A sharp (♯) means a shift upward and a flat (♭), a shift downward. We may illustrate by indicating the shifts which must be made when the keynote is D instead of C:—

Formal instruction should follow the first practice in reading. Here again the child might be taught the differences between different keys in a formal manner, and this method was formerly pursued, with the consequence that it was made so difficult that the young child could not learn it. All that is necessary at the beginning is that the child become accustomed to sing-

ing in different keys and that he learn to recognize the basic notes in a particular key as being situated on a particular line or space. A device[1] for helping to recognize where the keynote is situated is used in a series of books published by C. H. Congdon. In these books in the earlier melodies of a new key, the space or line upon which the keynote is situated is indicated by a faint orange line. The use of this method, whether or not accompanied by some such device, does not make the learning a perfectly simple affair, it is true, but it makes it very much easier for the child than it would be to enter into an elaborate explanation of the theory of the matter. In the case of different keys as in the case of the scale itself, formal instruction is necessary in order to complete the crude recognition which is gained without it. The time at which this instruction should be given is doubtless to be determined by the time when the child grasps clearly the fundamental characteristics of the scale.

Only formal instruction begins with the key of C. The key of C has been taken as a starting-point for convenience of description in this discussion, but it should not be inferred that the child is to begin his singing or reading in this key. The starting-point in the practice of reading or singing should be at the key or keys which suit the pitch of the child's voice. The starting-point in the formal instruction, as in this discussion, may be the key of C.

[1] This device is practicable only in the use of simple melodies in which there is no change of key, or modulation.

Rhythm

The most fundamental element in music is rhythm. We have taken melody — that is, the sequence of tones — as the first element of music to be mentioned, because it is the one which most readily attracts the attention and the representation of which is most prominent. As a matter of fact, however, melody is not the fundamental element of music. There is another element that precedes it in racial development and also in the development of the child. This is the element of rhythm. Music may in fact be regarded as a specialization of the rhythmical activities. Rhythm is used in a great variety of forms of action besides music. The music which is characteristic of primitive peoples consists chiefly of rhythm as illustrated in their drum-beating. Rhythm also is characteristic of many of our actions which we do not think of as at all related to music. Many forms of work are carried on in rhythm and can be done with the most economical expenditure of energy in this way. Many of the physiological processes, such as the heart-beat and breathing, are rhythmical in their nature. Speech itself is rhythmical, and this may be verified both in prose and poetry. In the case of music, the rhythm is somewhat more regular and is connected with the elements of pitch and tone.

Rhythm consists in the regular recurrence of acts divided into groups. In a rhythmical activity, such as

music or dancing, the individual acts which compose the entire chain of acts follow one another at regular intervals and are also arranged in groups each containing, in the simpler cases, a uniform number of single acts. Marching is a simple illustration. Each step represents the single element, and each step takes the same amount of time as every other step. These single steps are usually put into groups of two each by accenting by a drumbeat or the voice every other step, commonly each step of the left foot. The rhythm becomes more complex in dancing, but it can always be analyzed into these two elements, the regular succession of a series of steps or movements and the division of the series into groups.

Music has developed out of dancing. This illustration suggests the relation which dancing holds to music. Dancing was the more primitive form of expression and was developed as a part of the religious ceremonies of primitive people. In order to unify the dancing of a group of people and to intensify and emphasize the feeling of rhythm, the dance was accompanied by the beating on the drum and monotonous singing. In order to furnish more varied accompaniment, musical instruments were gradually developed and the songs gradually acquired more definite and complex melody.

Training in rhythmical movements prepares for music. It is necessary to recognize the fact that rhythm has a deeper seat than the music which is sung or played on an instrument. Rhythm which is only kept

by following the beat of some time-marker, such as the metronome, must always be mechanical, formal, and artificial. To be a significant form of expression rhythm must be felt, and it is felt when the whole body responds, and does in fact move in harmony with the tempo of the music. This makes it clear that a complete training in music must go back to the training in rhythmical bodily movements. The training in dancing, of the type which is béing introduced into the schools, is not only of value to secure bodily control and poise, therefore, but is also a valuable introduction to the appreciation of music.

Rhythm is represented by notes and bars. While the pitch of the tones is represented by the position of the notes on the staff, the rhythm is represented by appropriate signs, and the child has to learn to interpret these. The duration of a tone is represented by the kind of note which is used, and the length of each unit group of notes is indicated by the division into measures. The pupil has to learn to recognize the type of rhythm which is represented in the different kinds of time — two part, three part, four part, etc.; to hold the tones the length of time indicated by the notes, at the same time fitting them into the rhythm; and to indicate the divisions between measures, or the lesser divisions between parts of measures, by accent.

Formal instruction in tempo and its representation should come after some reading ability is gained. A

child may learn to give the proper duration to the various notes, and to give the accent at the proper place in the measure, and to associate these with the aspects of the printed score which represents them, in the same way in which he learns the meaning of the different tones in a melody. He learns to associate the elements of rhythm with the characteristics of the score which represents them at first by the incidental method. It becomes necessary in the course of time to call his attention particularly to these facts and to give him a clear idea of their meaning. Here again the ability fully to understand the complexities of the elements of music and their representation need not be developed before some ability in reading has been gained.

Harmony

Harmony rests on the recognition of consonance and dissonance. There is still a third characteristic in highly developed music which the child can appreciate to a greater or less extent, but which is not so fundamental as those of melody and rhythm. This characteristic is harmony. Certain tones, when sounded together, give persons who have a sense for musical harmony a pleasurable experience. Others when sounded together produce displeasure. The psychology of harmony and disharmony is not entirely clear. All we know is that, for most persons, certain combinations are pleasing and others are not. Some combina-

tions also are more pleasing than others. Among those which are pleasing we recognize different degrees of harmony. The octave, for example, is recognized as a combination in which there is almost complete fusion of the two tones. Certain other intervals — for example, those which are separated by four steps or by three steps, as the combination C and G, or C and F — make what is called "perfect consonance," because no disharmony appears when they are sounded together. Others separated by intervals of two steps, as C and E, or E and G, make what is called "imperfect consonance," because some disharmony is felt when these chords are sounded. It is not necessary to go into the various psychological theories which have been offered to explain the difference between harmony and disharmony, particularly since the whole matter is now put in question by certain newer types of music, in which the previously accepted laws of harmony are violated.

There are individual differences in the appreciation of harmony. The ability to appreciate the difference between harmony and disharmony requires higher development than that required to recognize melody. Children differ considerably in their ability in this matter, and some are capable of a higher development than others. Harmony is, of course, involved in part singing, and it is necessary that when children engage in this phase of music they should be able to appreciate the simpler phases of harmony. It will probably, how-

ever, never be necessary to go into the explanation of the matter in any detail with them, and so we may leave it without further discussion.

Tone quality

Another refinement which is involved when the training in singing is carried beyond the mere ability to strike the correct notes in singing melodies, or in singing parts in simple pieces, is called " tone production." This involves not merely the ability to sing a certain note, but also the ability to produce it in such a way that the quality of the voice is pleasing. A tone possesses pleasing quality when it is rich and smooth rather than thin or rough. The tone quality of a voice or an instrument is called the " timbre " of the tone. This is involved in the appreciation of the higher or more developed phases of music, and it is necessary that the child gain an appreciation of it and the ability to produce good tone if he is given specialized training which either looks toward professional equipment or the higher degrees of amateur development.

While there is not time or opportunity to give much individual attention to tone quality in the singing of children in the school, considerable improvement can be brought about by calling attention of the group as a whole to the quality of their voices and by setting up standards for imitation. Something may also be done in this direction indirectly by requiring the children to maintain good bodily posture.

Individual and age differences

Individual differences in musical capacity should be taken account of in selecting children for training. As has been incidentally mentioned, there are large individual differences in children's ability in music. An attempt has been made to measure these individual differences, and to determine their significance for the child's ability to profit by training. C. E. Seashore has measured the ability of children to distinguish between tones of different pitch and has laid down certain rules as to the amount of training which children should receive who have different degrees of discrimination. Seashore has the following to say on this point:[1] —

> The capacity for the appreciation of music is partially inborn and partially the result of training. Thus, in judging the quality of an instrument or voice, the expert hears and observes differences and peculiarities that entirely escape the untrained ear; and all differences in the so-called quality and timbre of tone are reducible to pitch. But such hearing represents a complex process of interpretation, which can be mastered only after extensive training. The mere detection of pitch difference is, on the other hand, a simple process requiring only the slightest amount of training.

With reference to the bearing of the differences of pitch discrimination upon the amount of training which the child is capable of receiving, the author has the following to say: —

> Suppose we find four children of equal age, advancement, and general ability sitting together, and one has a threshold,

[1] *Psychological Monographs*, vol. 13, no. 1 (December, 1910), p. 54.

for pitch discrimination, of ½ vd., another 3 vd., another 12 vd., and another 25 vd. [The symbol *vd.* means the number of vibrations which are produced in a second.] They are to have singing lessons. How can we group them properly for this period? Nine years ago the author proposed the following classification as a tentative measure [*Educational Review*, June, 1901]: —

Below 3 vd. . . . May become a musician.
3–8 vd. . . . Should have a plain musical education. (Singing in school may be obligatory.)
9–17 vd. . . . Should have a plain musical education only if special inclination for some kind of music is shown. (Singing in school should be optional.)
18 vd. and above. . . . Should have nothing to do with music.

The significance of these figures may be grasped by the fact that 16 vd. represent one half of the difference between middle C and D, the next note above it, or about one half step in the scale.

Results of tests should be applied with discrimination. These should be taken as tentative standards, and should be interpreted as indicating the degrees of ability below which children should not be forced to undergo training, rather than as rigid limits marking off children who are not to be given training even if they are willing or desirous of receiving it. In some cases, as in that of a teacher, it is desirable to make an especial effort to develop some ability in music even though the native endowment in this direction is limited.

Instruction may begin early. With reference to the changes in age in discrimination, Seashore has the following to say: "In a bright child with a good ear,

the physiological limit can be established for all practical purposes as early as the age of five." This means that the child can very early distinguish as clearly between the different degrees of pitch as can the adult. It does not mean, however, that the young child is as capable of carrying tunes, or of appreciating the more complex features, such as harmony and the composition of a piece. These abilities depend not merely on the ability to distinguish between tones, but also on the ability to carry in mind the relationship of a large number of tones to one another, and this ability is only of gradual development. The significance of Seashore's statement is that the child's musical development may begin at an early age. The development of the higher forms must be gradual.

REFERENCES

1. C. H. Congdon: *The Congdon Music Readers*. Chicago.
2. C. H. Farnsworth: *Music in Education. Cyclopedia of Education*. Macmillan (1913), vol. 4.
3. C. E. Seashore: *The Measurement of Pitch Discrimination. Psychological Monographs* (1910), vol. 13, no. 1, p. 54.

CHAPTER VI

SPELLING: FIXING OF ASSOCIATIONS

Learning to spell is a form of memorizing. Memory is the means by which those experiences which we have had in the past are reproduced in our present experience. The particular kind of memory which we call "memorizing" consists in reproducing certain definite associations which have been formed in the past. Memorizing, then, consists in so making associations between ideas, or between words which represent ideas, that these words or ideas may be recalled in the same order in which they were originally formed. Spelling may be regarded as exactly the same sort of learning. In the case of spelling, the associations which are formed are between the successive letters of a word, and between the word thus spelled and the meaning. This might very properly be called memorizing, and is one form of memorizing, though the term is usually used to designate the formation of associations between words rather than between the letters of a word.

Accurate spelling is demanded chiefly in writing. The problem of learning to spell is one which arises in connection both with reading and with writing, but it would not be an acute problem if the child never had

to write. He would be able to recognize words with sufficient precision if he did not know their exact spelling. Only in certain cases of the close similarity of words would any difficulty arise. When it becomes necessary to write words, the problem of how to spell them becomes important. The child might express his meaning fairly intelligibly without learning to spell accurately, and until recent times even educated people expressed their meaning through writing in a satisfactory way without spelling with any high degree of uniformity. The extreme care which is now taken to spell a word in a certain conventional way is a relatively recent development. Formerly people did not even spell their names consistently. The school is compelled to take the situation as it is, and to recognize that a person who does not spell words in the conventional way is regarded as uneducated by other persons, and that such a one is likely to meet with difficulty in his business, professional, or social life.

The associations in spelling

The ability to spell may be based on associations between movements or between elements of perception. While the problem of spelling first becomes acute as a result of the need of writing, it nevertheless is closely related to the problem of learning to read. The reason for this is seen when we examine more closely into some of the associations which lie at the bottom of the ability to spell. Association must always be

between certain ideas or experiences, and one type of experiences which may be associated consist of motor activities. In the case of spelling these activities may be the writing of certain letters, and some persons can spell the more difficult words only when they write, or at least when they imagine themselves to be writing. We call this type of association a motor association because it consists of connections between movements. Or the association may be between movements which are made in the pronunciation of the letters of a word. This was the common method of spelling in the earlier days of spelling instruction. At the present time, when the letters are not so much emphasized in learning to read, this is not so common. There may be an association, also, not between these forms of movements but between ideas or percepts, based upon the perception of the successive letters. Some persons can spell most readily when they see a word or think of the way the letters of the word look. It is with this type of association that reading has most to do. The impression which is made through carefully scrutinizing the word as it is read will then be of help in forming the association between the letters. A fourth type of association, which is used by many in assisting them to spell words, is between the letters as they are heard. One may either pronounce the letters himself and hear them as he pronounces them, or he may hear them pronounced by another. This is of the same sort as the association between the letters as seen, since it is an

association between the objects perceived rather than between movements. This form of association does not arise in connection with reading, but only in connection with special training in spelling.

Spelling instruction should be closely related to reading and writing. The fact that a necessity for spelling arises in connection with reading and especially with writing, makes it necessary, if spelling is to be taught in such a way as to have the most meaning for the child, that it shall be closely connected with reading and writing. This does not mean, as we shall see in a later section, that there need not be special drill given to spelling; but it means that the words which are given in the spelling drill should be those which the child uses also in his reading and writing. It means that, although it is necessary that the child give special attention to the spelling of words, the words to which he gives special attention should not be out of relation to his experience and meaningless to him, but they should be related to the rest of his thinking and of his school work. In choosing the words which the child is to learn to spell, we must take into account those words which he shall have use for after he leaves school. Investigations have been conducted to determine what these words are. When these have been found, it is not sufficient merely to introduce them into the spelling lesson, but they must also appear in the other work of the child, in his composition, in his reading, and so on.

Why may not the child spell entirely by sound? It has been said that learning to spell consists in making connections between the acts of writing the letters of a word, or speaking the letters, or in making associations between the letters as they appear on the page, or as they sound when spoken. The question may be asked why it is necessary to make these associations. Theoretically, as we saw in the chapter on reading, the letters of a word represent the sound of the spoken word. If this is the case, why, then, may not the child merely form once for all the association between a sound and its corresponding letter, and thus be able to spell any word which he can pronounce?

Partly because pronunciation varies. There are several reasons why this is not sufficient. One reason is that the variation in the pronunciation of words makes it impossible to be always sure, from its pronunciation, how a word is spelled. An illustration of this fact is given in a book by Owen Wister in which he ridicules spelling reform. According to the story, a number of enthusiasts for spelling reform met in a convention, and attempted to agree how certain words should be spelled so that the spelling should conform to the pronunciation. One Southern delegate started a riot by insisting that *courthouse* should be spelled *c-o-a-t-h-o-u-s-e*, this being the way in which he pronounced the word.

Partly because the same letter often represents a variety of sounds. It was noticed in the chapter on

reading that the alphabet which is used in written English, in common with most of the European tongues, has been inherited from the Greeks and Phœnicians through the Romans, and that it does not completely fit the English language. The same letter often represents a variety of sounds. Consider the sound of *a* in *bay*, *bat*, *ball*, *bar*, *ask*, *autumn*, and *ribald*; of the *s* in *sit* and *business*, of the *c* in *cat* and *receive*. Such variations as these make it impossible to infer from the spelling of a word how it shall be pronounced.

Chiefly because the same sounds are often represented by a variety of letters. The chief reason, however, why the spelling of English words cannot be learned merely by the connection of sounds with letters is that the language is not spelled as phonetically as it might be. We do not spell the words so as to correspond as closely to pronunciation as we might even with our imperfect alphabet, and with the variety of dialects and modes of pronunciation which exist. Various spelling-reform associations have made us familiar with the great variety of cases in which the same sound is represented in several ways or in which the same spelling represents a variety of sounds. The following examples may be taken at random. Consider the different ways in which the sound of long *o* is represented in the following words: *so*; *sew*; *row*; *though*; *bureau*. So long as our spelling remains in its present unphonetic condition, and we have no means of knowing how long it may so remain, there is no

possibility of a child's learning how to spell many of the words of our language from their sound. It becomes necessary, then, for him to learn merely by forming arbitrary associations between the letters.

Methods of learning to spell

So far as possible the child should learn to spell from sound. Before going on to a consideration of the way in which these arbitrary associations are formed, we should keep clearly in mind the fact that in the case of those words which can be spelled from their sound, the child should learn to spell them through connecting the sound with the spelling. The principle here is the same one which is found to hold in the case of memorizing. It is found that the material which is to be memorized should be given a meaning, as far as may be, and that rote memorizing should be reduced to as small an amount as possible. Similarly, we may say that so far as the child can get a clue to the spelling of the word from its sound he should do so. Furthermore, although there is a variety of ways in which the same sound may be represented, yet the number of ways is not so large but that the child can learn what they are. The child's problem in spelling a word which is not phonetically spelled is not to learn a purely arbitrary spelling for it, but to learn which of several ways of representing that sound is the one which is used in this particular word. Spelling is not a purely arbitrary affair, and in the case of a great

many words the spelling can be deduced from the pronunciation. Because the correspondence between the sound and the spelling is not complete is no reason for failing to make the most of the degree of correspondence which does exist.

All available helps from related words, rules, or principles should be used to make spelling rational. We may say, as a general principle, that wherever the spelling of a word can be related to some inclusive rule or fact, or may be connected with the spelling of analogous words, this should be done. Such relationships may be found not merely between the sound and spelling, but also between the spelling of different words which have similar meanings. For example, the word *laboratory* presents a difficulty particularly in the syllable which is not accented — the second syllable. The fact that *o* is used here may be connected with the word *labor*. When the child once sees this connection, he will easily remember how the longer word is spelled. Even more or less arbitrary rules, if they serve to bring the spelling of a word under a principle and make of it not merely an arbitrary affair, will be useful, such as the familiar rule regarding the order of *i* and *e* in *receive*, *believe*, *weigh*, etc.

To fix the arbitrary association the incidental and the drill method may be contrasted. After we have done all we can to find a connection between the spelling of a word and its pronunciation or to establish other meaningful associations, there remain a certain

number of purely arbitrary associations which have to be made. There is no royal road to spelling. In the formation of these associations, which are left over after we have exhausted all the logical associations possible, there are two general methods of procedure. The educational opinion and practice have been divided upon which of these two methods is the more effective and economical. The first method consists in allowing the child to learn to spell words merely as they happen to come up in the course of his reading or of his writing. Some special attention may be given to the words as they appear, either in order to anticipate and prevent a child's spelling a word wrong, or in order to correct a wrong spelling after he has made it; but no attempt is made to give the child a drill in the spelling of words in a separate period, or to select series or lists of words upon which repeated drill is to be given. This is the incidental method. The drill method, which is opposed to the incidental method, includes the characteristics which have been described as being neglected by that method. It takes its name from the fact that it applies the principles which have been found through psychological investigation to produce in the most economical and effective manner the associations which are desired to be formed.

The investigations of Rice and Cornman give support to the incidental method. Before attempting to present more minutely the advantages and disadvantages of these two methods, we may sketch very

briefly the experiments which have been performed in order to compare the efficiency of the two methods in actual practice. In the late nineties, J. M. Rice startled the educational world by showing, through a survey of the results of spelling from a large number of cities, that a great deal of the time spent in spelling exercises was wasted. This was inferred from the fact that many cities in which a smaller amount of time was spent secured as good results as did those in which a much longer time was taken for spelling. This investigation was followed up by another by Cornman, in which the experiment was made of entirely eliminating the spelling period from three Philadelphia schools, and comparing the result of the incidental method used in these schools with the results of the spelling drill as used in the remainder. In the incidental method, as used in these three schools, considerable attention was given to spelling by calling the child's attention particularly to the difficult words which he was likely to misspell, and by collecting the mistakes which he made in his written work. This experiment showed that the three schools which used the incidental method secured as good results as did the others which employed the ordinary drill.

Wallin's investigation turned the scale in favor of the drill method. These investigations seemed to settle the question in favor of the incidental method. There were, however, several weaknesses in the reasoning leading to such a conclusion, and these have been

shown to be fatal to it in a more recent investigation by J. E. W. Wallin, at Cleveland. The Cleveland schools had for years used a carefully worked-out form of spelling drill under the direction of Assistant Superintendent Warren E. Hicks. Wallin found that as a result of this drill a degree of efficiency was reached by the children of the Cleveland schools which far exceeded that obtained by either the drill group or the incidental group of the Philadelphia schools.

The incidental method is superior only to a poor drill method. The results found by Wallin demonstrate that the superiority in economy of the incidental method, which was used in the three schools in Cornman's investigation, was due to the fact that the drill method which was used in the other schools was not as efficient as it might have been. On the other hand, the incidental method in the three experimental schools was probably raised to its highest point of efficiency. The average percentage of correctly spelled words in the city was about seventy, with two or three per cent advantage in favor of the schools which used drill. Wallin, on the other hand, found a percentage of correctly spelled words, with words of the same character, of about ninety-four. This clearly indicates that if the drill method is correctly applied, it may give results more than twenty per cent better than those obtained by the incidental method.

The principles of the drill method

We turn now to a consideration of the principles of drill which are necessary in order that the method may be most effective. These principles may be derived from the principles of habit formation in general, which is of the same character as drill, and therefore are not to be thought of as bearing solely upon the teaching of spelling.

The first principle of spelling drill is sufficient repetition. As we saw in the chapter on handwriting, the cardinal principle of the formation of arbitrary associations is repetition. If we make an association once, or several times, but not a sufficient number of times so that it is fairly well fixed, we have wasted in a large measure the time and effort required to make these insufficient repetitions. A number of repetitions spent upon a small number of words, sufficient in number to make the ability to spell these words permanent, will give a definite result. If the same number of repetitions are spread over so many words that none of them are learned thoroughly, they are largely wasted. This is what happens in the case of the incidental method. Words are only learned as they occur in the child's reading and writing. It is then only a matter of chance that the same word occurs with sufficient frequency so that the child learns it in the most economical manner. It is to supply this want that drill is introduced.

The second principle is adequate attention. The next principle of drill has also been met in the consid-

eration of writing, namely, that repetition made without giving attention is of little value. This principle is sometimes put first and is designated by the term "focalization." When there is focalization of attention, the child has his mind called sharply to the thing he is doing. In the case of spelling, this means that when he is learning to spell a word, he is thinking primarily of its spelling, and not of its meaning, or of the form of the letters as he writes them, or of some other fact connected with it. When the child learns to spell words incidentally in connection with reading or writing, his mind is divided between the spelling and the thought which he is getting from the reading, or which he is expressing in his writing, and perhaps the form of the letters, as has already been suggested. Spelling drill, in which the words are studied particularly for the sake of knowing how to spell them, calls the child's attention to this one fact or aspect of the word, and therefore brings about the condition of focalization.

The third principle is the avoidance of wrong associations. A third principle which has also already been met with, and which is particularly important in spelling, is the avoidance of wrong associations. The drill method attempts to do this in a systematic manner by anticipating the words which the child is likely to misspell and giving him special practice in their spelling. An investigation by S. A. Zook[1] has shown

[1] This investigation is reported in an unpublished master's thesis which is on file in the library of the University of Chicago.

that it is desirable, in addition to calling the pupil's attention to the word which he is likely to misspell, to call his attention also to the particular parts of the word where it has been found the children frequently make errors. For example, it was found that there was frequently confusion as to what part of the word the *c* comes in, in the word *scissors*. If this is particularly emphasized in teaching the child the word, many errors will be avoided. This, of course, does not mean that the wrong spelling is to be suggested, but that the correct spelling of the difficult part is to be emphasized.

Drill may be made effective for spelling in connected writing. One of the criticisms which has been made of the drill method is that it does not enable the pupils to spell the words which they have learned in their spelling lessons when they write spontaneously, to express thought. The results are said not to carry over from the spelling lesson to the rest of the pupil's work. This defect, however, can readily be avoided, as was proved by Wallin's investigation, by giving the pupils practice not merely in spelling the words in a column test, but also in using the words in sentences, and in connecting the spelling with the meaning. This does not violate the principle of focalization, since the spelling is the chief subject of consideration.

Drill need not be uninteresting. Another criticism which has been made is that the "spelling-grind" is dull and uninteresting, and is deadening to the spon-

taneity of the pupils. If the drill is properly conducted, this criticism does not hold, and the belief that drill is uninteresting to the child rests upon a false interpretation of those things which give him pleasure. The child is certainly interested in perfecting his ability in various forms of skill and spontaneously practices doing many things which involve a large amount of repetition and arduous practice. If the work is suited to his capacity, and if he can be shown the results, and, when necessary, if some external stimulus is given such as is furnished by a contest between different rooms or between different schools, as was done in Cleveland, there is no difficulty in making such a drill of great interest to the child. As has been suggested in previous chapters, it is desirable not merely to compare the child's record with that of others, but also to give him some record of his own progress from time to time. Through this means it will be possible for him to see the results of his efforts, even though he stands toward the bottom of the class on account of deficient native ability in this particular line of work.

The best method of presentation has been studied. A good deal of experimentation has been carried on to determine what is the best avenue of presentation of words and of letters in order that they may be learned most economically; and also to determine whether persons differ in the avenue of approach which is best suited to their individual needs. In attempting to answer the first question, classes have been taught by

presenting the words to them through vision, that is, by writing on the board, and the results have been compared with the results from other classes to whom the spelling of the words was given orally; and from others who have seen the words written and have also pronounced them; and others who have seen the words and also have written them, etc.

Writing is the most effective single method, but a variety should be used. In general it has been shown that writing the words and seeing them written is the most effective form of presentation for all pupils taken together. Hearing the letters of the words pronounced and pronouncing them, which is the old spelling-contest method, is not so effective. This is probably due to the fact that the pronunciation of the letters is not closely connected with the sound of the word, and is, of course, not so closely connected with the way in which the words are spelled in actual use, which is the process of writing itself. In general, however, to use a variety of methods of presentation is more effective than to use one alone, and since there is no reason why one needs to be chosen alone, the safe course is to present the words to the child by writing, by having him write them and pronounce them slowly, and possibly by having him say the letters as he writes them.

A variety of methods takes account of individual differences. Another reason why a variety of modes of presentation is desirable is that there is possibly sufficient difference in the type of minds of different pupils

to make it easier for some to learn through hearing and pronunciation than through seeing and writing. The differences among pupils in this respect have probably been exaggerated, and we do not know with certainty to what degree they exist. It is probable that there are only a few who represent any extreme type. However this may be, there is no harm in presenting words in a variety of ways, and if there are pupils who represent extreme types, this will be to their advantage.

Summary. We have attempted to bring out the more general psychological principles which are involved in spelling, and which may be illustrated in spelling. In particular, spelling illustrates the drill method, and the principles which have been found to have a place in spelling are also appropriate wherever we are concerned with the formation of more or less arbitrary associations, and where we are dealing with learning in which there is a gradual progress instead of a solution of the problem by means of the understanding.

REFERENCES

1. O. P. Cornman: *Spelling in the Elementary School.* Ginn & Co. 1902.
2. L. P. Ayers: *A Measuring Scale for Ability in Spelling.* Russell Sage Foundation. 1915.
3. W. F. Jones: *Concrete Investigation of the Material of English Spelling, etc.* University of South Dakota Bulletin. 1913.
4. J. M. Rice: "The Futility of the Spelling Grind." *The Forum* (1897), pp. 163 and 409.
5. J. E. Wallin: *Spelling Efficiency, etc.* Warwick and York. 1911.
6. ~~H. Suzzallo:~~ *The Teaching of Spelling.* Houghton Mifflin Company. 1913.

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CHAPTER VII

HISTORY: EXTENSION OF EXPERIENCE THROUGH IMAGINATION

General account of imagination

Learning in history and geography depend upon the imagination. Although the same mental processes are used in various subjects, certain of them seem to be more particularly involved in some of the school subjects than in others. If we compare the study of the two subjects which are about to be considered, history and geography, with some of the more elementary subjects, such as reading and writing, we find that they are distinguished by the fact that the child is introduced by them beyond his immediate experience. Reading and writing as they are used carry the child's mind beyond his immediate experience; but in *learning* to read and write the child is dealing primarily with material which is presented in perception. In history and geography, on the other hand, he is required to live through experiences which are remote in time from his own life, or which are distant from him in place. In history he learns about the lives of people in the past, and in geography, he learns about places and people who exist on other parts of the earth than that in which his home is situated. The mental process

by which he is able thus to carry in mind the thought of distant or remote events, persons, and places, we call imagination.[1]

Through imagination present institutions can be explained by reference to their history. The school, for example, has certain peculiar characteristics. The division of the school period into eight years for the elementary school, four years for the high school, and four years for the college, has a definite historical background. The methods of study which are pursued, the subjects which are included in the curriculum, and a variety of other matters, are to be explained by reference to more or less remote past events.

Imagination connects our immediate life with events taking place in distant parts of the earth. For the explanation of many of the events which we observe to be going on about us, we must be able to create in our imagination persons and things which exist in the present time, but in remote places on the earth. As this is being written, much of the conduct of residents in the United States is governed by the war which is being waged in Europe. We are continually creating in imagination the scenes of the European conflict and considering its bearing on our lives. The world has

[1] Imagination is the mental process by which experience is extended beyond the immediately present surroundings. This is done in its simplest form in memory. But imagination also extends experience much more broadly so as to bring within the scope of our thought objects or events which have never been experienced by the individual in the form in which they exist in his thought.

come to be so much of a unit through means of transportation and communication that the use of geography is an everyday affair.

Imagination which may be present in memory prepares for freer imagination. A form of imagination exists in memory. When we picture to ourselves the face of a person whom we know or recall the sound of his voice or the pressure of his hand-clasp, we make use of a memory image. The first step in the ability to think over or to live over in our minds that which we have never experienced consists in thus living over our own past experience. In this way we first free ourselves from the immediate present. In this the human being is rather sharply distinguished from the animal. So far as we know, the animal is pretty closely bound to the sensations or perceptions which he receives through stimulation by objects which affect his sense organs. We have no evidence that the animal thinks clearly of experiences it has had in the past, although its past experiences affect its responses, through giving objects various kinds of present meaning.

Sensory experiences may be revived in imagination. The human being, as has been said, may revive in his experience various kinds of previous experiences. The simplest of these are the sensations or simple perceptions. We can call to mind the appearance of the house which we saw hours or days or years before. We can call to mind, more or less clearly, the sound of the voice of persons whom we have known, or the sound of

music which we have heard. The sensations of sight and hearing are the ones which can be most clearly reproduced in our imagination, but to some degree we can recall the experience of touch and possibly of movement, smell, taste, and temperature.

Images of words are prominent in thinking. There is a particular form of imagination which deserves special mention. We may recall to the mind not merely the appearance of physical objects or of the experiences which we get from contact with them, but we may also call to mind the sound or the pronunciation of words. Words may be represented to us through the sound of some particular person's voice or through their appearance in writing or in print, or through the movement sensations we get in speaking. The particular form of the image does not matter. The essential fact is that ideas are represented by images of words. Many ideas are of such a nature that we cannot think of them most clearly by recalling any sort of object, but rather by recalling statements in terms of language. Take the law of gravitation. The word itself does not refer to a thing which we have seen or heard or felt, but to an idea, and the recall of the idea is through the recall of the word. Or, if we go further and represent to our mind the law of gravitation, — that bodies are attracted to one another according to the product of their mass and inversely according to the square of the distance, — it is clear that the words by which the idea is expressed are the chief embodiment of the idea.

Take another case. We express the idea of relationship of a person to his country by the word "patriotism"; but although patriotism may have certain symbols, such as the flag, or the ruler of the country, and so on, these symbols do not stand for the idea with the same degree of completeness as does the word. Very much of our thinking, then, is done in words by saying over words internally, or in the imagination. Even our thinking in regard to concrete objects is accompanied by the presence of images of words in our minds. We may then distinguish between concrete imagination which has to do with objects, and verbal imagination, which has to do with words.

The importance of words in thinking must not be overlooked. This fact that verbal imagination is an essential means to all our higher and abstract thinking should prevent us from misinterpreting the objections to verbalism in teaching or learning. There is a danger that the child shall learn to repeat words in parrot-like fashion without an understanding of their meaning. This is worse than useless. But to set words and their use over against thinking, and to imply that we must get rid of words in order that we may think, would be like getting rid of our feet as a preliminary to walking. We must be sure that the child has the experiences which give a meaning to words so that he shall not merely put words together into phrases and sentences; but we must realize that one of the important results of education is just this acquirement of meaning by

words, so that they can be interpreted and used intelligently. As the child grows older, we find that he learns to use more and more verbal imagination, as distinguished from concrete imagination. While the older person thinks somewhat in the images of things, yet they tend to become less prominent, and he represents his thoughts more largely in words.

There are wide differences in imagery among different persons. Not only are there these differences between persons of different ages, but there are also marked differences between persons of the same age in the definiteness and clearness of their images in general and in the preference which they give to images of different kinds. Some persons are able to recall very clearly such an object as the breakfast table, as was found in an investigation by Galton. Others can obtain only a very dim image of the articles on the breakfast table. Some can recall more clearly things which they have seen, others things which they have heard, and others even the movements which they have performed.

Methods of teaching should be adapted in some measure to these differences. We must to some extent take account of these differences, although not to so great an extent as has sometimes been thought. The way that these facts may be applied was mentioned in the discussion of spelling. It is difficult, however, to be always sure to what type a person belongs. He is not able to tell us himself, unless he is a psychologist, and

there are no simple tests by which we may determine the matter. We may apply these facts in two ways; first, by so arranging the work that it will appeal to a variety of forms of imagination in the children, and second, by allowing a child to do a thing in the way in which he seems to succeed the best. This does not mean that he should be allowed to choose the kind of work that he wishes, but rather that he should be allowed to adopt the manner of doing the work which seems to suit him best.

Free imagination begins by the modification of memory. We turn now to a description of the different grades of imagination and of the ways in which the different kinds are used in the study of history. The simpler type of imagination, as has been remarked, is that in which we live over the experiences which we ourselves have had in the past. By this means we break away from the present. A person may live over the events of the previous day, or a day in the past month, or the past year. When he does this, he soon begins to do more than merely to live over his past life. He begins to dwell upon certain phases of his experiences more than others, and it has been discovered that the more pleasant experiences tend to be retained longer than the unpleasant ones. Furthermore, he begins to change the character of his experiences somewhat. Part of this is due to reading into the past things which have happened since. Part of it is due to putting together things which were not to-

gether in our past experience. So in a variety of ways we take liberties with our past life, and modify it, either because of the frailty of our memory, or because of a tendency in our nature to dwell upon certain experiences rather than upon others.

The next step is to construct objects in imagination different from those which have been met in actual life. This modification paves the way for an entirely different kind of imagination, or rather one which may depart so widely from the first stage of the merely reproductive imagination that it becomes radically different from it. To take our illustration from the life of a child, a child may first recall to mind experiences he has had with an object, as, for example, a dog. After he has become somewhat accustomed to this procedure, he may follow a story which is told him, relating to a dog, by supplying the picture of the dog which he knows. He thus uses his past experiences to fill out and make concrete events which are being related to him. As he acquires wider experience with dogs, he is likely to use, not the image of some particular dog, to make a story about a dog concrete, but an image which differs from particular dogs of his acquaintance and is constructed for the purpose of the story.

Imagination uses old material, but combines it in new ways. This illustrates a general principle regarding imagination. Imagination does not create anything absolutely new so far as the material of thought

is concerned, but enables us to rearrange into new combinations or into new forms the materials which have been furnished to us by our past experience. We may create these forms in our imagination as a means of following a story or description which is made by another person, or we may spontaneously create new combinations, as when we try to invent a new machine or the plot of an original story. When we merely follow the lead of another, the process may be called "constructive imagination," and when we spontaneously form in our mind events or objects which have not been present in our past experience, we term the process "creative imagination." The form of imagination in which we merely live over our past experiences may be termed, in contrast with these, "reproductive imagination." The distinction between these three is not always entirely sharp, but they may serve to represent types which ar distinct in their main characteristics.[1]

The great stimulus to the development of imagina tion in the child is language. The use of words b other persons is a continual challenge to the child form in his mind ideas which correspond to th words. This he does by the recall of his previ

[1] Reproductive imagination is the type which is illustrate memory. Constructive imagination is involved in the stud history and geography, in which we form in our minds ideas corresponding to definite requirements which are imposed upo minds from without. Creative imagination is involved in i or in literary productions.

experiences, or by anticipating in imagination experiences which he will have in the future. He hears older persons talk about the happenings of the previous day, and this causes him to recall them also to his own mind. He hears discussed in the family circle the plans for the next day or the next week, and this leads him to look forward. The conversation of others creates a stimulus to the formation in his mind of ideas which have not been present at all in his past experience, but which may be formed by putting together elements out of his previous experience.

The child's imagination is little hampered by restrictions set by natural law. A child's mind is particularly apt in creating with great freedom new combinations in his imagination. He has not had sufficient knowledge of the ways in which different things may be expected to combine, or of the laws which govern their combination, so that he is particularly hampered by such conditions. He can put things together in his imagination which are impossible for the adult, from the fact that the adult is aware of their improbability. To the child few things are improbable and still fewer are impossible. The myths and fairy tales with which he is regaled are appreciated by him because the combination of events and of things which are related in them does not strike him as absurd. That Santa Claus can drive his reindeer through the air, and can come down through the chimney, are, to be sure, in contradiction of any events which he has experi-

enced; but his sense of law and order is not yet sufficiently developed to make it difficult for him to believe that such things can happen.

The child cannot guide his imagination according to definitely prescribed conditions. While the child's imagination seems to be more active than that of the adult because of his greater freedom from the restrictions which are imposed by natural law, he is inferior to the adult in the kind of imagination which is demanded in history. As we shall see shortly, it is necessary, in order to understand history, to construct an idea of the physical surroundings, manner of life, and mental attitudes of persons who are remote from ourselves in time and customs. This means that we must break away in a measure from the restrictions of our accustomed experiences, and also that in constructing the new conception we must faithfully follow the prescriptions which are laid down for us by the historian. It is in the ability to follow definite prescriptions so as to gain a faithful historical conception that the child is deficient.

Imagination need not be fanciful, but may be based on facts. The distinction which was made in the preceding paragraph suggests a characteristic of imagination which is in danger of being overlooked. We often think of imagination as something which is purely fanciful, something which is unreal or untrue to our experience. The child is particularly adept at this type of imagination. The other type which requires

the construction in our mind, according to certain definite principles, of conditions which are different from those in which we live, is equally genuine, and is one of the most important requirements of scientific thinking. The geologists create a different world from that which exists at the present time by taking a few bones and fossils of plants and building an environment about them by the application of the laws and principles which we find to be present in the world about us. They determine what the conditions of life and the appearance of the earth must have been in past ages, and are able to draw a picture or to construct a model which represents in its general features what these conditions were. The historian is able to do the same thing for the life of people who have existed centuries ago. This is the type of imagination which is not at all fanciful, but which is determined by fixed laws and principles.

The types of imagination employed in history

Imagination of concrete objects

History involves the creation of persons or places in the mind's eye. Some kinds of historical appreciation involve certain types of imagination, and others involve others, and what these are we shall have to consider more particularly. In the first place, we must use in history the same type of imagination which we use in the appreciation of fiction. When we tell the

child about historical persons, he has to create some sort of image of these persons in his mind, in exactly the same way as when we tell him about persons who only exist in fiction. This type of imagination enables the child to appreciate the earlier and simpler forms of history, which we may designate the "Once-upon-a-time" kind. It is not necessary that the child should locate the persons in definite time, or in definite order with reference to one another. By means of this kind of historical narrative he becomes acquainted with some of the important personages with whom he will later deal in a more systematic way. The only difference between this kind of history and fiction is that in the one case the characters have actually existed, whereas in the other they have not.

Familiarity with the biography of historical persons prepares for the appreciation of their historical relations. As has been said, it is worth while to use historical personages in the stories which are told the child, because it prepares him to weave these personalities into the web of historical account which will later be related to him. It will then be easier for him to follow the later accounts since the elements are familiar to him. This type of history is suitable to the child from the time when he can first appreciate it, that is, from about the third year, until he is about nine or ten years of age.

Temporal imagination

Complete understanding of history includes placing events in a time order. In order that the child may progress beyond this stage, it is necessary that he be able to place the historical personages in past time, and in definite order in time with reference to one another. This rests upon the development of another type of imagination, which we shall call "temporal imagination." The stories relating to primitive life of mankind, which are used in the primary grades, do not give the child a true historical notion of the way in which civilization has developed from primitive life down to the life which we find about us at the present time. They prepare him for a later appreciation of such development, much as do biographies. That they cannot give the child a notion based on the conception of the succession of different stages of development in time may be seen from the fact that, according to the Binet Scale for testing intelligence, the average child does not until his sixth year appreciate the difference between morning and afternoon. It is to be said that this scale is too easy for the average American child, but it is such a far call from the simple appreciation of the difference between morning and afternoon to the understanding of development which has extended through thousands of years, that it is inconceivable that the primary child gets much more than a series of pictures from tales of primitive life. That

these tales are suitable for the child from other points of view is, of course, not called into question.

The idea of long stretches of time is related to the perceptions of short intervals. The child gets his idea of the passage of time and of events, as being more or less distant in the past or future, as an outgrowth from his immediate experience with time. He first must learn to distinguish between events which are near at hand in the past or the future, and then, after time has come to have a meaning for him, he can apply it to the arrangement of more widely separated and farther distant events. What we call "temporal imagination," or the idea of time, is undoubtedly based to some extent on what may be called the "time sense," or the immediate perception of the passage of short intervals of time. Experiments with adults upon the time sense indicate that it is subject to a good deal of variability according to the circumstances. If a person is anxiously expecting some event, such as the arrival of a friend, the time will seem very long. If, on the other hand, he is not definitely expecting an event, and is engaged in some occupation which absorbs his attention, he may greatly underestimate the passage of time. The overestimation of the passage of time is expressed in the common proverb, "A watched pot never boils."

The child is deficient in time sense. Both experimentation and ordinary observation indicate that the child's time sense is much less accurate than that of

the adult, and that the variations due to the manner in which the mind is occupied are much greater in the child than in adults. The child is deficient in the ability to imitate rhythm, and grossly misjudges the lapse of time. A child becomes very impatient when waiting for an expected event. On the other hand, if he is absorbed, it sometimes appears that he has no sense of time. If a child is sent on an errand, and if his attention is attracted by some occurrence on the way, he does not realize the length of time which he spends in stopping to watch the interesting occurrence, and arrives home late.

The recognition of long periods of time depends partly on the time sense and partly on the observation of external events. The idea of time, or temporal imagination, is undoubtedly based to some extent on this fundamental time sense. The time which elapses between breakfast and the midday meal and between this and the evening meal is appreciated vaguely by means of the direct experience we have of the passage of time. In order that the time idea shall be more definite and extend to longer periods, it is necessary that we observe other facts and use them to mark off definite divisions of time. The way in which the larger divisions of time are marked off indicates that we are thus dependent upon external events, and not merely upon our own appreciation of the passage of time, in order that we may get a unit of time and may apply it in the estimation of the longer periods. The year, for

example, represents changes in the seasons, the month marks the phases of the moon, the day the rising and setting of the sun, and so on. The smaller divisions of the day, except those between midday and morning or evening, are not distinguished sharply unless we have watches or clocks which give us the means for marking off the hours. The inferiority of the young child in the recognition of periods of time, then, depends partly upon an undeveloped time sense and partly upon the fact that he has not been led to observe the signs which mark the divisions of time, and to use them to set in order his memories of what has happened in the past or his anticipation of events in the future.

The child learns to distinguish periods of time through his everyday experience. As indicated in the preceding paragraph, the child first makes the distinctions which are called to his attention by the practical needs of his life. He will distinguish between morning and afternoon, when during the forenoon he is promised something which is to occur in the afternoon. Again, during the afternoon his recollection of the forenoon and his discrimination of that period from the rest of the day may be stimulated by conversation about something which occurred then. The distinction between day and night is called sharply to his attention both by the fact that the sun goes down in the evening and rises in the morning, and also by the fact that morning and evening mark important turning-points in his experience, in that he goes to bed

at night and gets up in the morning. Similarly, he is led gradually to make the distinction between yesterday and to-morrow through recalling the events of yesterday or anticipating those of to-morrow. The reward or punishment which he may have received for acts done previously is one method by which this distinction is called to mind; and in a similar way, the promise and fulfillment of results of his actions of to-day call to his attention the coming of to-morrow.

Longer periods are recognized in the same way. The recognition of larger divisions, such as that of the week, is produced in a similar way by the events of his life which occur at the end of the weekly period. When the child goes to school, his attention is called to the fact that Saturday is marked off by the holiday which occurs on that day, and that Sunday brings a still different manner of life, such as putting on his best clothes and going to church or Sunday School. The division into still longer periods of time is called to his attention by the presence of other holidays or vacations, or events which occur in a periodic way. His birthday, for example, helps to bring to mind vaguely, and later more clearly, the notion of the year.

Number is necessary to the clear idea of long periods of time. Any near approach to an exact notion of the longer periods of time requires that the child shall not only remember and anticipate in a general way, but that he shall make definite the duration of time inter-

vals through the use of number. We can, perhaps, get an appreciation of an hour or a day merely by the direct experience of the passage of time or by the time sense; but the appreciation of a month in any definite way means that we have a definite number idea of the meaning of thirty as a multiple of one day. In the case of the longer stretches, such as a year, it is obviously impossible to appreciate them merely by the direct time sense. We may, therefore, assume that the child gets only a very vague and indefinite notion of longer time until he has a sufficiently clear idea of number to put the time intervals into definite numerical form.

The abstract notion of even the longer periods may be made somewhat concrete by observing natural events. The appreciation of time through the observation of such events, as well as by means of the number idea, rests upon forms of experience which the child gets only gradually and does not attain at an early age. All of these forms of analysis of the temporal imagination indicate that it is a matter of gradual growth on the part of the child, and make it clear that the degree of clearness which is necessary for the appreciation of definite historical sequences is not to be looked for until the child has been in school for at least three or four years. Even then the development is not completed, but is only thoroughly started.

First historical narratives should be simple. These conclusions, if well founded, can only lead to the posi-

tion that the historical narratives which are designed for the child must at first be simple and easy to follow. In the first place, they must not cover very long periods of time. Even the account of the life of a single individual is beyond the grasp of a child at first. Furthermore, the various series of events must not be complex. It is easier to follow the life of one individual than it is to follow the lives of several individuals who are contemporary with one another. It is easier to follow the history of one section of a country than the parallel histories of all sections. It is easier to follow the history of one country than the history of several countries which are interwoven in their relationships with one another. The difficulty, however, does not depend so much upon the breadth or extent of the fields covered as upon the minuteness with which the train of events within a certain field is followed out. The life of one person may be very complicated if the relationships of this person to others, and to the events of the age in which he lived, are followed out in full detail. On the other hand, the history of a country may be made very simple by the selection of a few representative events which portray certain phases of its development. We may say, then, that the history which is designed for a child must be simple, and furthermore, that it is easier to simplify the history of one country than it is of several related countries. Therefore, it is best to begin with the history of a single country, and after that has been followed and

the child has got some conception of its development, it is time to relate the history of other countries to it.

Begin with recent history and pass from that to remote events. The problem then arises as to which country should be treated at the beginning and as to the way in which the history of the country which has been chosen should be developed. The psychology of the development of the idea of time, as it has been outlined, throws some light on this problem. We have found that the child first recognizes the differences of time which are immediately in the past or the future; that he begins with his own personal experience and reaches out from it before and after. If we follow this same process in the early history which the child is taught, we should begin with his own country, with which he has immediate experience, and should teach the child comparatively recent history at first, rather than begin with remote past time and work forward. We cannot, of course, carry out the principle of beginning from the present and working backwards, in the full and strict sense, as one may reverse a moving-picture film; but we may follow the psychological order by relating the history of events which are near the child and gradually extending them to those which are more remote in the past.

This method connects history with present life. When we do this, we make of history something which is of immediate connection with the child's present experience, and which lends significance to the prob-

lems which arise in his own life and throws some light upon their solution. History may be regarded as an account of the origin of those facts and those beings which are in existence at the present time. In tracing the origin of those facts or beings, we begin with their immediate past and then work gradually farther and farther toward their more remote antecedents. If we begin, in teaching the child, with the remote antecedents of present-day people, or with the remote causes of existing physical objects, he loses the connection between the present results and the remote past cause. He then fails to get the conception of history as having a direct and important relationship to his present-day affairs.

History involves spatial imagination. Historical events are, of course, not only related in time, but are also related to one another in space, since they take place at particular localities upon the earth. The child is required, therefore, in any complete appreciation of history, not only to be able to put the events together in a certain time order, but also to create in his imagination the geography of these events. This form of imagination will be treated particularly in the chapter on "Spatial Imagination," or "Geography."

The historical sense

The child at first fails to make allowance for the differences between past and present life. When the child is first told of persons or places which existed in

the past, he has a strong tendency to think of them as like the persons with whom he has been familiar in his own experience. He merely transfers in imagination those things, those scenes, and those persons which he has met in his own life to a previous period, and puts them into the story which the history relates, modifying them as may be necessary to suit the events. In his early acquaintance with history, the child does not recognize the difference in the physical surroundings, or the difference in beliefs and mental attitudes, which characterized people of past time in contrast to those of our own time.

The difference in physical surroundings is appreciated first. The ability to appreciate the difference in physical surroundings is the easier one to acquire, and the one which the child can acquire in a more or less superficial way by the use of pictures and other concrete means of representation. Still, it requires a fairly good degree of ability in constructive imagination to picture to one's self clearly the houses, the landscapes, and the costumes of the people of past times.

The appreciation of differences in mental attitudes is difficult even for adults. The difficulty of realizing to one's self the difference in physical surroundings is very much less than that which attaches to the appreciation of the differences between the mental attitudes of persons of past time and those of our own contemporaries. Even our ancestors of one hundred years or one hundred and fifty years ago differed from us so

radically on certain questions that it is difficult for us to understand how they could have held some of the beliefs that were common. To take an example from our own country, a justification of the attitude of the colonial settlers toward witchcraft is inconceivable to us. It is even becoming difficult for us to understand how there could have been any real difference of opinion on the question of slavery. If an adult finds it difficult to project himself into the mental life of a past people, even of those who are his immediate ancestors, it is an impossibility for the child to do so with any degree of completeness.

The historical sense extends the attitude of tolerance to persons who lived in the past. This type of imagination, which has been called the "historical sense," is in reality merely an extension of the process which the educated adult carries on when he gains a proper degree of insight into differing points of view of other persons with whom he comes in contact. Various groups of people in the same society differ in their attitude on all sorts of questions. A hidebound adherent to one political party can as little conceive how any one can conscientiously uphold the foolish principles of the opposing party as can the average person of this age conceive how a rational person could have believed the things which our ancestors believed. What we are considering here is merely the extension to wider differences of this same tolerance and understanding. This extension must be built upon somewhat more

abstract material. In understanding the attitudes of historical persons we have to create for ourselves their lives and attitudes on the basis of written records and traditions, rather than on the basis of contact and conversation with them. We believe that an increase in tolerance is being attained by modern civilized people, and similarly, the historians tell us that the fully developed historical sense is distinctly a feature of modern scholarship

The historical sense has little possibility of development before adolescence. The child can begin at a rather early age to gain in the appreciation of the difference in physical surroundings of past people, but the full development of even this type of historical sense is not consummated early. The higher type of historical sense, which deals with mental rather than with physical facts, is probably very slightly developed before adolescence; and then it has only a gradual growth. It appears from the recorded observations and investigations of children that the ability to appreciate the motives of others, to understand motives which are different from one's own, and to judge of one another's conduct by considering motive as well as actions, is an attainment which is not reached before adolescence. If this is true, it is futile to attempt to discuss with the child to any great extent the differences in point of view between ourselves and people of the past, and to explain them on the basis of the growth of morals or of customs, as can be done with the older pupil.

The grasp of historical development

Two other forms of mental ability are prominent in the appreciation or study of history in its full meaning, although they are not, strictly speaking, forms of imagination. In order to round out our account of the mental processes in the study of history, however, we may include a brief discussion of them.

History attempts to explain events by previous conditions and events. The first form of history in the mind of the child is merely a narrative, that is, a series of events occurring in time. These events are related to one another, but not in such a way as to make prominent the idea that the later events grow out of the earlier ones as their cause. The degree to which the study of cause and effect can be carried on in history is a question on which there is a division of opinion. Some students of history have attempted to give a very minute account of the course of human events as illustrating the law of cause and effect. The discovery and formulation of general laws with reference to historical development has been attempted. Although one may not agree with the extreme form of this view, yet historians do occupy themselves with such questions as are being considered at the time of this writing with reference to the European war. Not only historians, but all persons who have any interest in the matter, are desirous of knowing what the causes of such a tremendous conflict are. Was the war caused

by the machinations of a few individuals, or by the ambitions and desire for territorial expansion on the part of a war party; or was it caused by the expansion of peoples, or their desire to gain for themselves the good things of the earth; or was it caused by an inevitable conflict of races; and so on? In order to gain any light at all on such questions, we must study events which have led up to such a consummation as this, and we must give these events an interpretation. As in the case of the historical sense, it is probable that children gain little appreciation of this phase of history until they reach the adolescent period.

Critical examination of sources

Another phase of the study of history has not to do so much with the understanding of historical facts after they have been gathered as with the method by which they are gathered and determined upon in the first place. Ancient historians discriminated very little between tradition, which had no foundation in fact, and reliable sources of evidence. We may say that what is regarded as the scientific method of obtaining facts is the most prominent characteristic of modern historical scholarship.

The examination of historical sources is an examination of witnesses under difficulties. In order to ascertain the reliability of records which purport to be historical, it is necessary that the same procedure be carried out, in a modified and more difficult form, that

is taken to find out the truth regarding contemporary facts. Courts of law have developed an elaborate procedure by which facts in a disputed case may be discovered. We know that even in the case of current events, it is difficult to be sure what is the truth and what is fiction. The examination of witnesses and the criticism of their testimony are now being studied in a scientific way. We may regard historical records as testimony of witnesses to be examined in the same way, with the added difficulty that they cannot be cross-questioned. The possibility of prejudice or of the distortion of evidence because of personal interests are matters which have to be considered. When a historian attempts to determine what are the facts in regard to the conduct of a war, for example, he can find conflicting evidence in newspaper reports which are published in the different countries. It is his business to decide what the truth is by comparison of the different sources of evidence.

Critical examination of sources is too difficult for high-school pupils. It is sometimes the fashion to use a good deal of source material in the teaching of history to young people. It is questionable, however, whether they are able to carry on the very difficult and complicated sort of examination of sources which is necessary to get anything of value from them. It is perfectly possible and altogether desirable to make historical accounts concrete and vivid by introducing certain forms of source material which are agreed upon

as reliable and which do not need to be examined critically. But this is a different matter from the examination of source material for the purpose of determining upon its reliability. It is probably safe to say that practically none of this kind of examination of sources is suitable for the pupil of high-school age, and that little of it can be done to advantage before the later years of college work.

Summary. In summary it may be said that the study of history has its chief value and purpose in the enrichment of the imagination of the child, by extending his experiences into the antecedents of the life in which he finds himself. This extension of his experience backwards enables him the better to understand the life in which he participates, and because he can better understand it he can better take his part. As a result of the study of historical development he may grasp something of the trend of present events. He may avoid repeating old errors and may help to carry forward the community life in the direction of progress.

REFERENCES

1. J. R. Angell: *Psychology*, chap. 8. Holt, 1910.
2. H. E. Bourne: *The Teaching of History and Civics*. Longmans, 1903.

CHAPTER VIII

GEOGRAPHY: EXTENSION OF EXPERIENCE THROUGH IMAGINATION

In the introduction to the previous chapter, it was shown that our experience may be extended beyond the confines of our own immediate surroundings, in the imagination of events in past or future time, or of those events which are distant in space. It is with this latter kind of imagination that we are concerned in this chapter. The means by which the location of places upon the earth becomes organized in our minds is the systematic presentation of the facts which constitute geography. It is not necessary to make a sharp distinction between geography and the simpler forms of astronomy, since we can understand the occurrences upon the earth only by understanding its relation to the sun, the moon, and the other heavenly bodies.

This treatment does not disparage physical and commercial geography, which are important. In considering the phase of geography which deals with the location of places upon the earth, it is not necessary to assume that other forms of geography, which have become prominent in the schools in recent times, are not to be included in the study of this subject. Physi-

cal geography, as an account of the way in which the earth has developed, and of the varieties in physical structure of the continents and the other features of the earth's surface, is a desirable part of the study, since it gives the child a conception of the origin of the earth in its present form. Similarly, the study of the natural products of various parts of the earth's surface, and of the effect of altitude, or of latitude, or of other physical conditions upon the life of men and upon their occupations, — all of which we include under the head of "Commercial Geography," — is highly appropriate as a subject to be taught in the school.

The other phases of geography belong to natural science. While recognizing the importance of these topics, we may still hold to the belief that the fundamental conception which underlies the understanding of these other matters is the localization of the different parts of the earth. This means the ability to picture in the mind different localities, their distance from each other, and the direction which routes from one place to another have with reference to each other. Since the other two phases of geography are by nature related more closely to natural science than they are to location or place geography, we may include them in a general way in the consideration of natural science and confine ourselves here merely to what we may call "spatial imagination," or localization of places upon the earth.

Growth of spatial imagination or orientation

The development of spatial imagination is gradual. The ability to picture to one's self the location of a variety of places on the earth with reference to one's own location, and with reference to one another, or the more complicated form of imagination by which one pictures the relationship of the planets to the sun or to one another, and their movements, is subject to development through certain stages, as is temporal imagination. The child comes only gradually to the fully developed ability in this as in other forms of learning.

The idea of direction is based on the sense of bodily position and movements. As in temporal imagination the child first gains the ability to think of events in the immediate past or future, so in the case of spatial imagination he first becomes able to think of objects or places which he does not see, but which are in his immediate neighborhood. This ability is the outgrowth of the localization of sound which the child gains when he hears a sound behind him and turns his head in its direction. The child learns in the first year to do this quite accurately. The later development consists in imagining the source of a sound without looking around, and then in imagining the location of other objects which are behind him, or which are out of the range of his vision, without turning around to see them. In the earlier case one thinks of a sound very much in the terms of the change in bodily position

which one would take in order to turn toward the sound. This is one of the fundamental facts which is characteristic through all forms of our ability to locate direction. We think of the direction of a place with reference to our bodily position, and the direction is represented in our mind in a large measure in terms of the change in bodily position, as in turning the head or in pointing, which would bring us into a different relation toward the place.

The localization of directions is gradually extended to more distant objects. The development of this ability to localize places which are not seen is gradually extended. The child learns not merely to place in his imagination objects which are in the room or which might be seen, but also those which are at a greater distance and which cannot be seen from his present position. After a few years, at perhaps four or five, the child comes to have a fairly definite notion of direction of buildings or of streets which are in the immediate vicinity of his home, and he can point in the general direction of familiar objects which he cannot see.

Orientation with reference to one's self leads to orientation of places with reference to one another. The whole process of localization we call "orientation,"[1] and it has become evident that orientation be-

[1] Orientation is the localization of objects in imagination either with reference to one's self, to one another, or to fixed, standard directions.

gins with the localization of places near one's self, and that it has a close connection with one's sense of bodily position. Orientation is in the first instance a sense of the position of other objects with reference to one's self, or the sense of one's own position with reference to other objects. This primitive form of orientation, which we may call, perhaps, "personal orientation," or "variable orientation," is a step in the development of a wider form of orientation which is more fixed in its nature. In this more highly developed form the child learns to think of a number of places as having a fixed relationship in space to one another. After the child has learned the direction of different buildings and streets with reference to himself when he occupies various positions among these places, he builds up a notion of their directions and distances with reference to one another, which does not depend upon his position in the group. He learns to think of a street, with the buildings which are located on it, with its relationship to other streets. This depends on the ability to take a detached view of a region as though it were seen from a distance.

Objects are further oriented by being put into a system of fixed directions. This development of a more extensive and a fixed orientation usually takes place through relating different positions or directions with reference to the position of some fixed object or of some fixed direction. Whether or not this is always the case in the young child, at least it is the means by

which the more complicated forms of orientation are developed. The early mariners used the north star as a point from which they reckoned their positions, and the directions of places toward which they were sailing, and we still use the north and other points of the compass to enable us to organize the great variety of directions and locations which we meet with in our different experiences. The use of some such fixed point of reference, then, is almost necessary for development of any high degree of orientation. Here, again, we find that bodily position and bodily movements play a large share in the development of orientation. After one has determined upon the point of reference with which the various places are to be located, he can form the ideas of the location of these places by facing the standard direction, and then, either actually or in imagination, turning toward the different individual places.

The correction of false orientation makes us conscious of the nature of the process. Such a system of orientation as has been described is built up so gradually, and, in many cases, so early in life, that it is difficult to recall the steps through which it has passed, or even to realize that the system exists. It is brought forcibly to mind, however, in those cases in which we form a wrong orientation. If we go to a new town and immediately get the directions correct and realize the position of the town with reference to north, south, east, and west, the whole system of orientation is built

up without our being conscious of it. But when we have made a mistake and have got these directions wrong from the start, we find that the correction of the mistake is very difficult. We become conscious of the strength of the bonds between the various points or directions of the place and the general points of the compass when we attempt to change them, in order to relate the buildings and streets in detail to a different position of the cardinal points.

False orientation has to be readjusted piecemeal. The great confusion which results in such a case of mistaken orientation and the process by which it is necessary to correct the mistake are instructive as to the way in which orientation is built up in the first place. We find in such cases that it is not possible to correct the mistake we have made once for all, that is, merely by recognizing that we have reversed the directions of north and south, or have thought of north as east; but that it is also necessary to change the direction and locations in our mind of every part of the city which we have formerly thought of with the wrong orientation. It becomes necessary, so to speak, not to turn about our idea of the city as a whole, but to rearrange it piecemeal, to readjust every small section, with which we have become acquainted, by itself, until finally the whole becomes readjusted.

Orientation is in part built up by relating parts individually to cardinal points. This fact indicates that in our first acquaintance with the different parts of a

place, we adjust each by itself with relation to the cardinal points of the compass, and we think of each part thereafter, without being fully aware of it, as related to these cardinal points. Our idea of a house is not merely the idea of its location in a certain part of a city, but also of the house as facing east or west or north or south. When we have thus become accustomed to the direction of the facing of the house, we have to readjust our ideas with reference to this house itself and not merely with reference to the place as a whole. At least, while it may not be necessary to carry out the process in every detail, it is necessary to go into a considerable amount of particular readjustment in order to correct the original false orientation.

The child gradually gains a conception of the cardinal points. Just as an adult, in coming to a new place, gradually becomes accustomed to think of the parts with which he becomes acquainted as having a certain position in reference to the points of the compass, the child gradually builds up, as well as his idea of the directions of particular places, his notion of the cardinal points of the compass themselves. This is probably done first with reference to some one place. The child becomes familiar with the fact that the sun rises in a certain direction and sets in the opposite direction with reference to his own home and the neighboring streets and buildings. He then, perhaps, acquires the idea of north with reference to the rising and setting of the sun and to the stars, etc.; and after these general

notions have been acquired in connection with his home town, he applies them to other towns and then makes the ideas more and more general.

Imagination extends sight experience, as sight extends touch experience. The development of orientation, as it has been outlined, consists in the formation of ideas of the direction of objects or places with reference to one's self, the development of the idea of distances between objects and of the direction which one is from another, and of directions with reference to a standard direction. We have now to consider the refinement of ideas of distances and directions between places and of the way in which these ideas are built up in some detail. When a person forms in his imagination the idea of places with which he is familiar and the distances and directions between them, he is extending his experience in much the same way that we extend the experience which we get through handling objects and through our direct contact with them, when we not only touch them, but also see them. The blind man is very limited in the extent to which he can organize objects into a well-ordered whole. It is said that his ideas of objects in relation to one another, so far as they are clearly worked out, are limited by the amount that he can span with his arms, that is, by the extent of space from which he can get simultaneous sensations. We can organize objects in a better manner when we experience them simultaneously than when we experience them one after another. We may think

of the imagination as extending one's experience of space beyond that which is apprehended through perception, or through seeing objects, in much the same way in which sight extends the experience which we get through touch. To put it in another way, if a person lacked the ability to form an organized conception of different places in his imagination, he would be limited, in comparison to the person who possesses the ability to develop orientation through his imagination, in much the same way that the blind man is limited in comparison with one who sees. This does not mean that the imagination supplies the place of seeing. The blind man can fill out through his imagination what he lacks through his vision. It serves merely as an analogy to illustrate the fact that we have different kinds of spatial recognition which are different in the range of objects or places which they can include. In the first place, we have the ability to recognize, through touch and movement, the form of objects and the position of objects which are within a very small range. Broader than this is the sense of sight, which enables us to recognize simultaneously wider ranges of objects. Still broader is the imagination, which gives us an orientation among places so widely separated that they cannot be perceived at the same time, even by sight.

Imagination may be stimulated by the sight of a large district from a high place. The transition from sight to imagination in the development of orientation

may be bridged by taking the child to a high place, so that he can get a sight of a larger number of places than he ordinarily sees. This enables him, by means of the broad range of vision, to obtain a unified view of those places with which he has already become familiar. A child may be taken to a tall building or the top of a hill and given the opportunity to survey the neighboring streets and buildings. He will then get an idea of the whole, which he can hold in his imagination while he walks about upon the ground and while his vision is limited. He may be stimulated thus to keep in mind the location of places in his imagination by questions regarding the directions or distances of places from the place on which he stands, or by questions regarding the direction of one place from another, or their distance apart.

Maps and globes

Maps are symbols which are interpreted by the imagination. This practice will prepare the way for the child to form the idea of the relation of places to one another, even though he has not seen them together. This transition from sight to imagination is greatly facilitated by the use of maps. The child may draw his own map of a region which he sees from a high place, or the location of the places upon the map may be pointed out to him as he sees them from the eminence. The child is thus brought to the understanding of maps as signs or symbols of the relation of

places to one another. When he has learned through this means what a map is for, and what it refers to, he is able to use a map as a means of guiding his imagination in understanding the relationship of places which are so distant that they cannot be seen at the same time.

Maps are necessary to insure accuracy in spatial imagination of large areas. It is possible to get only a very vague and rough idea of the location of widely separated places unless a map is used. Suppose that one should try to get an accurate idea of the relative location of three places by traveling from one to another. The three places might form the three apexes of a triangle. In order to form an accurate idea of their location, it would be necessary to keep in mind not only the direction in which one is traveling in going from one to another, but also the distances. If an error was made in either the distance or the direction, it would introduce an error into the notion of the positions of the places with reference to other places and with reference to one another. On the map, due to the fact that it is drawn to scale, the relative distances as well as the directions are represented to the eye. In attempting to get a clear notion of a region, one must either consult a map which has been made of it or make a map by means of compass and a measuring device. It is difficult for one who has been brought up to the early use of maps to imagine to himself the sort of orientation which a person would have who had

never used maps; but it seems clear that a person who does not have the assistance of maps will require a much longer time to become oriented in a region than one who uses maps.

After being first used in close relation to the concrete, maps may be used abstractly. After the child has learned the meaning of maps by associating them with the regions with which he is familiar, he may learn to use them without referring directly to the actual world which is about him. It is not necessary that a person in planning a railroad journey shall at every moment think of the direction or the distances which are represented on the map as related to the place which he is occupying. He may be in one city and be planning a railroad journey which is to start from another city. In that case if he applies the map to the concrete world, it must be by imagining himself in the other place. It is very probable that we learn to use maps in a highly abstract way for such purposes, but, as in a great many other cases, the meaning of maps will be greatly restricted if they have not in the beginning been closely related to the actual world. It is, therefore, highly desirable that the child should begin by a study of the region in which he lives, and that he should learn the meaning of maps with reference to this region instead of beginning with a map and attempting afterward to apply it to the real world.

Should the child begin to study his neighborhood or the earth as a whole? This raises the question which

is similar to the one which confronts the teacher of history, and that is whether the child studying geography should begin with those facts which are open to his observation and in his experience, and gradually work to those which are more remote; or whether he ought to begin with the larger, more general facts and then fill in the details by his later study. In the case of geography there is, perhaps, more to be said for the position that one should begin with the broader facts. We may say that the child can best understand the world when he realizes that it is a globe and that it revolves about the sun, and rotates on its axis, and so on. These facts are necessary for the comprehension of the seasons, for example. Furthermore, the child cannot understand the setting of the region in which he lives without knowing something of its relationship to the larger divisions of the earth.

Some home geography is necessary as a starting-point and basis for broader study. On the other hand, the general principle that the child's comprehension of general facts should be based on his experience of his immediate concrete surroundings holds in this case as in the case of history, — at least, up to a certain point. A map means nothing to a child unless he has first learned the relation of a map to the region with which he is acquainted. A globe means nothing to him unless he realizes that it is a representation of the earth upon which he finds himself. It is, therefore, necessary that he have at least so much of an acquaint-

anceship with his neighborhood that when he comes to study the more comprehensive symbols he shall know what they represent. This does not mean that he shall necessarily progress continuously from the narrower to the broader facts, but that he shall have a fairly complete experience with the narrower facts, so that when the broader ones are presented to him, they will have a clear meaning.

The existence of the earth as a whole and of the heavenly bodies is brought home to the child in his everyday experience. The case of geography is a little different from the case of history. In the first place, the rest of the earth and the other bodies which are related to the earth are presented in some measure to the child's present experience. He sees the sun and the moon and he observes the sun rise and set. He can see the outline of the shadow of the earth upon the moon. Furthermore, the other parts of the earth than that which is within his immediate experience exist simultaneously with the parts with which he is acquainted; and although he does not have direct experience with them, he meets references to them frequently. He may hear about a war which is taking place at some distant place, or be told of events which are occurring or have occurred in the past at other places. Perhaps he was born in another town from that in which he lives, or even in another country, and he hears these places talked about. And so, in a variety of ways, he learns indirectly about other parts of the world than

those which are in his immediate neighborhood. Again, the different parts of the earth are related to one another as a whole in a somewhat more intimate way than are the different periods of history. For these various reasons, then, it seems necessary to introduce the knowledge of the earth as a whole, and even of its relationship to the sun, the moon, and the other planets, as soon as the child has had sufficient concrete experience to give a meaning to the symbols which are used, — that is, to the maps and to the globe.

Care is needed properly to associate directions on the map with the cardinal points. A difficulty with the understanding of maps sometimes arises from regarding a certain side of a map as representing the wrong direction in the concrete world. By convention we use the top of the map to represent north. This association must, of course, be formed in the mind of the child. It is probable that if his attention is not called particularly to the matter he will naturally associate the directions on the map with the directions on the earth corresponding to it as it lies upon the desk. That is, if the child faces the east or even if the map is on the east wall, and nothing to the contrary is said to him, he will think of the top of the map as representing the east. We have no experimental facts on the matter, but observation by teachers seems to indicate that it is desirable at first, in order to form the proper association, that maps be placed so that the direction

on the map corresponds to the direction on the earth. As the child becomes more accustomed to the use of maps, he can readjust his position in imagination, but at first the connections between the symbols and the facts is somewhat difficult for him to make at the best, and it should be made as easy as can conveniently be done. For this purpose maps which are drawn upon the earth itself — for example, a map of the school-yard made upon the playground or in the sand-box — is the type of symbol which can be most easily associated in the mind of the child with the fact which it represents, and this is the step which is the most natural to take first.

Excursions accompanied by map-drawing assist the development of orientation. It goes without saying that excursions properly conducted greatly assist the child in the development of orientation. To be properly conducted, from the point of view of geography, means that the child shall during the excursion attempt to keep the directions and distances in mind, and that he shall draw a map of the region which is visited either during the trip or after the return. The fact that a person traverses a region does not mean that he has comprehended anything of the location of its parts. It is necessary that he himself actively observe the directions and distances in order that he may get any such notion. The leader is the one who knows where he is going and who can find his way about because he keeps directions in mind. A person who is not

attempting to lead becomes easily confused. There should, therefore, be some motive, such as the necessity of drawing a map, which shall lead the child to be attentive to this feature of the experience.

Summary. As was said at the beginning of the chapter, orientation is not by any means the whole of geography nor perhaps the most important part of it. It is the fundamental part, however, and without its proper development the other parts are much more difficult to learn properly. Physical geography and commercial geography are in the nature of natural sciences, and since we are not attempting to distinguish between the different branches of natural science, the general principles which are discussed in the later chapter on this subject will apply to them, and may be taken as sufficient discussion of them for the purposes of this book.

REFERENCES

1. A. Binet: "Reverse Illusions of Orientation." *Psychol. Rev.* (1894), vol. I, pp. 337–50.
2. C. H. Judd: *Introduction to Psychology*, chap. 6. Scribners, 1907.

CHAPTER IX

MATHEMATICS: ABSTRACT THOUGHT

Number an abstract mental process

In the mental processes and the forms of learning which have been discussed so far the responses have been governed in the main by the specific character of the objects which served as stimuli to them. In handwriting, the motor coördination is governed and directed by the specific form of the individual letters. In drawing, reading, and other forms of perceptual learning, the aim is to develop the recognition of objects in their particular character, and the facts which are significant concern the special characteristics of objects which distinguish them from others. Even in imagination, in which experience is extended beyond the concrete world which is present to the senses, the reference is still ultimately, in large measure, to the nature of the world as it is presented in perception. The characteristics of objects in which interest and attention center are their " real " characteristics, even though they are represented indirectly in the imagination. In number we have to do with a mental process which employs concrete experience only as a starting-point. As soon as the number symbols have acquired a meaning through concrete experi-

ence they become largely independent of it. How this abstract number idea develops we shall see in the following paragraphs.

The child first distinguishes between one and two or between more and less. The stages by which the child comes to the full number idea are gradual. His earliest idea of number is probably the distinction between one thing and two things, or is little more than the distinction between more and less. If he has two balls or playthings which are exactly alike, he knows when one of them is missing, and this appears to be the earliest manifestation of the recognition of the number of objects.

The number attitude is highly abstract. When the child uses number to the extent of counting, his attitude toward things undergoes a change. He regards things now merely as counters which are to be put together, and the interest is not in the things themselves, but in the fact that they can be put together in thought to make larger or smaller sums. This attitude toward things as being counters rather than having an interest because of their individual characteristics is a typical number attitude. We describe it in technical terms by saying that in number the attitude toward objects is an abstract one.[1] We are thinking only of one aspect of the objects, — that is, of their

[1] An abstract idea is one which does not merely reproduce the experience with concrete objects, but deals with one phase or aspect to the exclusion of the others. The use of language or other symbols makes possible such a high degree of abstraction that the symbols may have very little direct reference to the concrete world.

multiplicity, or of how many there are, — and we are not concerned, so far as our interest is the number interest, in their size, weight, shape, use, color, or any other of their properties.

The use of objects as counters is illustrated by the primitive herder. The use of objects as counters, which makes clear the abstractness of the number idea, is well illustrated by a practice which is occasionally employed by primitive people to make a record of the number of animals in their herd. Before counting has been sufficiently developed, so that there are enough number names to designate a good-sized herd, a primitive herder sometimes records the number of animals which he possesses by driving them through a stile and putting down a pebble for each one as it goes through. He then accumulates a pile of pebbles which corresponds in number to the herd. Each pebble represents one unit, and in the same way each sheep represents one unit; therefore, in this case the owner of the herd takes an abstract view of both the sheep and the pebbles. The sheep and pebbles do not resemble each other as concrete objects at all. They may not even be alike among themselves. The sheep may be large or small, young or old, with a plentiful supply of wool or a little supply; and the stones may be large or small, of regular or irregular shape and of various colors. In spite of the difference in all of these objects, however, they are all thought of as alike because they are regarded merely as counters.

Counting also illustrates the abstractness of number. The abstractness of number is also illustrated in counting. When the child gets sufficient command of the number names so that he can count readily, he is very much interested in counting all sorts of objects. He may count the buttons on his shoe, the chairs in the room, the houses in the block, or the trees in the yards, and so on. In thus applying the number names to a great variety of objects it becomes clear that he regards these objects, not from the point of view of their concrete character, — that is, of their use or their appearance or any other quality which leads one ordinarily to classify objects together, — but that he regards them in the highly abstract character of units or counters. In the application of any other names there is a more intimate similarity between the objects which are grouped together. Tables have in common the flat tops which we use for setting things upon; chairs have in common the fact that they are used to sit upon; and so on; but the objects which are counted have nothing in common but the fact that they are for the moment regarded as units and that they are put into a series of groups to which are applied the number names. The same number names may be used in counting any objects whatever.

Number names or counters are symbols. The illustrations which have been given of the simple use of numbers bring out one of the fundamental characteristics of numbers. In both cases a series of objects is

represented by a series of other objects which stand for them as signs or symbols. When the stones are used as representatives of the sheep, they are thought of only as symbols of the sheep and of merely the number of sheep. When the number names, "one," "two," "three," etc., are used to designate objects they stand as a sign or symbol of the number of the objects, each being regarded as one unit.

The early development of the number idea

There is a one-to-one correspondence between symbols and objects. In both these cases there is one sign or symbol for each object which is counted and there is one object for each of the symbols which is used. This correspondence is called a "one-to-one correspondence." It has been illustrated by Royce in the example of horses and their riders. If there are a group of horses and a group of riders, we pair them off by giving one horse to each rider and one rider to each horse. If there are any of one or the other left over, it indicates that there are more of one than of the other.

The child does not at first apply this principle in counting. This one-to-one correspondence is such a simple and fundamental matter that it may seem unnecessary to mention it. The child, however, does not grasp it when he first tries to count. If one watches the early beginnings of counting, one will see that the child frequently points to a number of objects to correspond to a single number name or uses several names

for one object. He has not grasped the notion that one name must be given to each object and that each object must have its name in order that the final number name shall represent the number of the objects which have been counted.

What is taken as a unit depends on the purpose in counting. It is worth while emphasizing that for the application of this one-to-one correspondence which is represented by counting, as well as the other forms of number into which it develops, it is not necessary that the objects used as units[1] be all of the same kind. We may count together any things which we choose to think of as units. In this case we do class them together sufficiently to put them into the same group. For instance, for the purpose of counting one may class together chairs, tables, beds, couches, etc., when the purpose is to count all the articles of furniture in the house. The influence which the purpose that the individual has in mind has upon the objects which he regards as units may be illustrated further in the case of counting all of the trees in a certain area, say an acre. In this case one will count every growing thing which is called a tree. One might, however, wish to count, not all of the trees, but all of a certain kind, as beech trees or pine trees. In this case the unit which is selected is of a more restricted nature. This is what is

[1] A unit is anything which for the time being is taken as single for the purpose of pairing, counting, grouping, or any of the number operations.

meant by saying that we may choose to call anything a unit which for the time being we choose to put into a certain group or class.

The idea of quantity implies similar units. The distinction should be made between number, in which the multiplicity of units is the chief idea and the concrete nature of the units is not important, and quantity, in which there is included also a different idea. We apply the idea of quantity to such matters as the amount of water in a tank, or the number of acres in a field, and so on. In other words, we use the term "quantity" to designate the number of identical units which exist in a certain total amount of material. To illustrate the difference from the example which was cited above, we find the number of trees by counting, and in this case a small tree counts for as much as a large tree. If we wish to find out the quantity of lumber in an acre of forest, however, we take as the unit a foot or a thousand feet of lumber, and calculate how many of these similar units there are in the grove. In this case every unit is the same as every other unit and the same number of units will always express the same quantity wherever it is found.

Early number is related to the fingers. The consideration of the act of counting, which is the application of the one-to-one correspondence by applying a series of number names to a series of objects, leads us to the question of the origin of the number names, and of the development of the understanding of number

names in the child. We have clear evidence that number names were often first developed through counting on the fingers. In fact, the word which we use to designate a single number symbol, "digit," indicates that the fingers had a close relationship to the early development of number. Further evidence that counting was developed through the use of the fingers appears in the decimal system, in which the number of the fingers forms the basis of grouping. We shall learn more particularly about this in a later paragraph.

The number names may originally have represented position in a series. In the character of the early names of specific numbers there is clear evidence of the origin of numbers from counting on the fingers. In the case of some primitive tribes, furthermore, the number names are the names of particular fingers and not merely of any of the fingers indifferently. This indicates that the fingers were counted in a certain fixed order and suggests that possibly the earliest number idea was one of a position in a series rather than of a group of objects taken all together.

Two illustrations of the use of names of particular fingers as number names follow.

The Hudson Bay Eskimos use the following number names:[1]

8. *kittukleemoot* = middle finger (of second hand).
9. *mikkeelukkamoot* = fourth finger.
10. *eerkilkoka* = little finger.

[1] L. L. Conant: *The Number Concept*. Macmillan, 1896.

The Jiviros of South America had the following names: —

5. *alacötegladu* = one hand.
6. *intimutu* = thumb (of second hand).
7. *tannituna* = index finger.
8. *tannituna cabiasu* = finger next to the index finger.
9. *bitin ötegla cabiasu* = hand next to complete.
10. *catögladu* = two hands.

Number names mean to the child, at first, position in a series rather than a group of objects. However this may be in racial development, it appears to be the natural order of development in the child. As the child is taught number by the common method of counting on the fingers, he at first thinks of each number name as being the name of one of his fingers. The name does not at first include all the fingers in the series up to the one in question, but merely the one which has been reached at the time when the name is spoken. In other words, the first idea that the child gets from counting is the ordinal rather than the cardinal idea. The numbers mean to him first, second, third, fourth, and so on. After he has learned to count the same series of objects in different orders, or to count other things than fingers, he gradually comes to realize that numbers represent a whole group of objects rather than one object which has a certain position in a series.

Our number system is based on a grouping by tens.

Before the child has proceeded very far in learning to count, he comes upon a very significant fact in our number system. This fact is that we have enough different number names to extend only a small way in the number series. We employ only ten different digits. As has been mentioned already, this is related to the fact that we have only ten digits on our hands. When we get to the end of this number of digits, we do not add other names, but begin over again, using a sufficient modification of the names to indicate that we are repeating the series a second time. When we have finished with the series a second time, we begin and go over it the third time, with a modification which is derived from the second digit, which indicates that this is the second time the series has been duplicated. The name and the symbol for twenty are directly related to two, and thirty to three, and so on.

This system of grouping reflects our inability to comprehend more than a small number of ungrouped objects. The limitation of the number of entirely distinct number names is due to another fact besides the origin of number in connection with the fingers. This fact is that the number of objects which we can hold in mind at the same time is limited. It is impossible for us to achieve an immediate and distinct recognition of a large number of objects unless we divide them into groups. This is precisely what we do by our system of number names. When we get beyond a number which constitutes the limit of a manageable

group, we begin a new group, and then designate it as being a second one. It is as though the primitive herder, in laying aside pebbles for his sheep, should put them into one group until he had got as many as he could grasp mentally at once, and then begin a second group.

The choice of ten as the unit of grouping is not the best possible. The name "decimal system" indicates the number of digits which are included in each group. It can be shown that it would be better, from the standpoint of the use of numbers, to take a different number for the unit group. Instead of using ten, twelve would be a more manageable number, because it has a larger number of factors. Ten can be factored only by two and five, whereas twelve can be factored by two, six, three, and four. It was an accident due to the fact that counting grew out of the use of the fingers that ten forms the basis of our system.

The grouping system facilitates the number operations. This division of numbers into manageable groups is of value not only because it enables us the better to grasp the meaning of larger numbers, and facilitates the systematic development of number names without extending the number of different names indefinitely, but also because it facilitates the processes of addition, subtraction, multiplication, and division. In adding the numbers twenty-three and forty-four we do not attempt to comprehend the combination of the two numbers as composed of single

units, but first add the single units which compose the fractional parts of the groupings, namely the three and four, and then combine the two larger groups; and finally obtain the result by combining the units and the larger unit groups. If the number is such that the sum of the single units come to more than ten, the excess above ten is put into the result as units and the group of ten is added to the other larger groups.

Grouping creates the distinction between higher and lower units. What the decimal system amounts to, as this illustration shows, is a method of using numbers to indicate the different kinds of units. By this means we may regard a digit as representing either a single unit or a group of these units. In our decimal system the group which can be represented as a higher unit is composed of ten of the lower units.

The distinction between higher and lower units is expressed clearly in our notation system. This distinction between a higher and lower order of units is most easily grasped by the use of the written numbers for illustration. The group system may be used for calculation even though it is not expressed in the notation by written numbers. The decimal system was used in Roman times in manipulating calculating machines similar to the abacus, while the written numbers were expressed in the clumsy Roman notation. A written number notation, which shall express the decimal system adequately, must represent in some manner the fact that a digit may stand for a single

unit or for a group unit, that is, for a lower or a higher unit. It is a commonplace with us that this is done by having only ten signs and using the same sign to express by its position either a single unit or a group of units. When we think of the application of the decimal system in calculating, then, we inevitably picture it in mind as expressed by numbers, and think of borrowing and carrying, for example, as they are worked out by our so-called Arabic notation.

Other experiences besides counting assist the child's development in number. These illustrations of the application of the decimal system have taken us somewhat ahead of the development of the child. We have seen that counting gives the child first the idea of position in a series and that he can by counting proceed to the recognition or the manipulation of fairly large numbers. He may by this means, it is true, recognize cardinal numbers, but the elaboration and perfection of this recognition can be carried out more easily by the addition of certain other forms of experience.

The idea of number develops out of experience with concrete objects. There has been considerable difference of opinion as to the kind of concrete experience which is best suited to the development of the idea of number in the child. It is well recognized that some experience with things is necessary at the beginning, to develop the early stages of the number idea, before it has become highly abstract. While the number idea

is not complete until it has become independent of the thought of any concrete things, yet it is developed first through the experience of the child with concrete objects.

Measurement of quantity, or grouped objects, may be used to give concrete basis for number. The two forms of experience which are commonly used to carry the child beyond the counting stage are measurements of length or of area or of cubic contents; and grouped objects, as marbles or balls or sticks. The measurement of objects means the dividing of an object into equal units and then the determination of the quantity of the object by calculating the number of the units into which it is divided. When the length of an object is to be measured, the calculation of the quantity is a simple matter of the application of a measure unit to the object. When an area is measured, the child may conceive it as composed of the small squares which may be used as units. The measurement of volume is still more complex. When sticks, or marbles, or balls on a frame are used, each unit is represented by a different object, and the combinations of numbers, their manipulation, and various operations may be illustrated by grouping these objects. The fundamental laws of number may be represented by the different ways in which a group of objects may be broken up into smaller groups, and then may be arrived at again through combining these smaller groups into the larger group.

Both methods may be used in combination to advantage. While divergent views may be held as to the value of the two forms of concrete experience which may be used in teaching the early numbers, it is not necessary that we choose either to the exclusion of the other. Each has its own advantages, and one may very well be made to supplement the other. The situation is not the same as it is in the case of some other differences of procedure in education which are mutually contradictory. It is probably desirable that more than one method be used in order that the child may the more readily handle the number processes independently of concrete material. If he learns with reference only to one kind of objects, he may find it necessary to refer to this one object in making his reckoning, but if he uses now one and then another he is more likely to become independent of both.

Grouped objects illustrate the decimal system particularly well. There are many number processes or ideas which may be illustrated conveniently by the method of grouped objects. One of these is the decimal system. The decimal system is primarily one of grouping. In it we set aside a certain number of units and regard them as units of a higher order. In using grouped objects this may be represented by counting ten and then setting these aside as a group, and then counting ten more and setting them aside, and then regarding as units the groups which are themselves made up of smaller units.

Number operations may be expressed as forms of grouping and regrouping. This grouping method may also be well used to illustrate the aspect of number which Judd has dwelt upon in his chapter on "Number" in *Genetic Psychology for Teachers*. It is there pointed out that the number operations may be derived from the fact that if a group be divided up into a variety of groups, these groups when recombined always give the original number. For instance, if we take a group of twelve objects we may divide it into two groups of six each, into three groups of four each, etc. After the division has been made, we may recombine the smaller groups again and always get the original twelve. In other words, the total number always remains the same whatever the grouping.

Grouping is a convenient means of teaching the fundamental operations. This is the basis of the understanding of the processes of division, multiplication, subtraction, and addition. The child by manipulating a group of objects may readily learn the various ways in which it may be broken up into smaller groups. For example, he may readily find that twelve may be broken up into three groups of four each, or four groups of three each, or two groups of six each, or six groups of two each. This is probably a more direct way of learning these facts than is measurement, though the same facts may be learned through the other method. Grouped objects are more easily ma-

nipulated than are units of length or of area, on account of the ease with which one may break up a group in a variety of ways and see the way in which the units may be recombined. The objects are more flexible and can be modified more readily than quantities which are suitable for measurement.

Measurement makes less prominent the units, but more prominent the size of the whole number. The use of measurement to make concrete these same processes may be illustrated. If a stick is measured into inch lengths, the child may count the inches two at a time and see that he has six, or may count the sixes and see that he has two. In the same way, he may divide the total length into three groups of four, or four of three each. Measurement makes more prominent the quantity as a whole than it does the individual unit, and it would seem to require a greater ability in abstraction on the part of the child to get a clear notion of the unit in this case, as compared with the use of distinct objects as units. From another point of view measurement is particularly suitable, since it calls the child's attention particularly to the size of the number as a whole, and is therefore useful in enabling him to see clearly the relative size of two numbers. If he has a stick twelve inches long and another six inches long, he may measure them and then, comparing the length of the two sticks with his eye, get a notion of the relation between the number six and the number twelve. We shall return to this

matter of the learning of the fundamental processes in later paragraphs.

Grouping is conveniently illustrated by the means of the abacus. The use of grouped objects to illustrate the simple number processes has been developed in the apparatus which is called the "abacus." This is a frame containing a series of parallel wires upon which are strung wooden balls capable of being moved back and forth. This instrument was formerly used widely in the United States, and there is a revival of this or some similar device at the present time. It has been developed in more complicated forms in Europe and is there used extensively.

In general we teach the meaning of numbers and their operations before teaching their symbols. The procedure which has been described assumes that the child learns to understand about numbers before he is taught to go through the formal processes of reckoning with figures. In the very beginning the child has some comprehension of the meaning of "one" and "two." He does, it is true, usually learn to count by a mechanical repetition of the number names before he has developed the understanding of the meaning of the different numbers very far, but, after this has been done, we commonly teach him the meaning of numbers through concrete objects before he learns to write them. Further, he learns the processes of addition, subtraction, multiplication, and division before he learns to use the plus, minus, multiplication, and divi-

sion signs; and he learns what it means to combine numbers and express their equivalence to a larger number before he uses the sign for equality. In general, the understanding of the process in concrete terms comes before the ability to express it in written figures and signs.

Teaching the symbols formerly preceded the teaching of the number fact. This order of procedure, in which the child learns the meaning of the number operations before he is taught how to express them, has not always obtained in teaching practice. In the early part of the nineteenth century a revolution took place in the method of teaching arithmetic in the United States. Up to this time the child had been first taught to write the figures and to go through the various operations with figures in a mechanical way. He learned as a matter of rote memory that four and five make nine, and learned by the same method to put the figures down in the proper way. It was not believed that it was possible for him to understand why he put down nine rather than eight or ten, or at any rate, it was not realized that it was a desirable thing that he should understand the process before he learned to carry it out.

Except in the beginning, mathematical operations are still frequently learned without an understanding of the reasons for them. The same difference in the method of teaching still runs through the higher branches of mathematics as well as arithmetic. It is a

possible mode of procedure, and one which is often actually practiced, to cause the pupil to learn the various algebraic formulæ, and to apply them, without any clear conception of what they mean. When the student comes to geometry, the demonstrations of the theorems are sometimes learned without any understanding of their meaning in terms of the figures which illustrate them. In arithmetic an illustration of the more complicated form of problems may be taken from the extraction of the square root. This may be taught in such a way that the pupils learn merely what to do next and how to proceed with the figures, in which case it becomes merely a sort of juggling feat. On the other hand, by the use of an appropriate figure, pupils may be taught, to some extent at least, to realize why they go through the various steps.

The question as to which of these methods should be pursued is one which arises early. When we come to the borrowing and carrying operations, as in subtraction and addition, and particularly when we come to long division, we have cases in which the form of procedure is rather complicated, and in which it may be plausibly argued that it is unnecessary and a waste of time to attempt to give the pupil an understanding of the reason for the performance of each step in the process.

In general, the more clearly the pupil understands what he does the better. The detailed answer to this question may be left to the discussion of methodology,

but at least we may say as a matter of general principle that it is highly desirable to develop in the pupil ability to understand what he does in contrast with the habit of carrying on a process in a mechanical way. The history of the development of number teaching has indicated that a pupil can be led to understand very much more than used to be required of him. While there may be details of some of the processes which it is not practicable to enable him to understand, he should at least know what he is aiming at, and have a general understanding of the reasons for the procedure which he takes. The question may be left open as to whether this understanding can be or should be made complete. It probably will be found that the procedure must be somewhat varied to suit the various capacities of the different children. Some, who are rather dull, may not be able to understand the procedure without too much time being spent or possibly even when an indefinite time is spent; and yet it is desirable that pupils should have a sufficient command of the simpler number operations to use them in practical life. On the other hand, for the pupil who is above average ability it is highly desirable that he shall acquire the habit of understanding clearly everything that he does. This is one of the most valuable attitudes of mind for a person who is to advance beyond the most elementary stages of intellectual achievement.

The understanding of the number processes must

be supplemented by drill in performing them. While there is danger of erring on the side of making the learning of the pupil too mechanical and of neglecting to cause him to understand the process which he carries on, there is also a possibility, and even danger, of erring on the opposite side. There is danger of assuming that after the pupil has understood the procedure, he has gained all that he needs to gain from that topic. This is altogether a mistake. We have, up to the most recent years, reacted against the use of drill as a means of developing in the pupil the ability to perform in an efficient way certain simple activities. We have already referred to this matter in connection with spelling. The same fault has been seen also in work with numbers. There has recently been a reaction against this neglect of drill and an emphasis upon the necessity of the pupil's being able to perform quickly and easily certain simple operations. We may merely repeat here the statement that to understand a process is not sufficient to enable the pupil to carry it out as he should. The effort to recollect how a process is to be carried on, or what the meaning of a problem is, makes the process slow and difficult. On the other hand, if sufficient practice has been undertaken so that as soon as the problem is presented the result comes to mind, or, if it is a longer problem, the mode of procedure comes to mind quickly, much time and energy are saved. There should be sufficient drill in all the processes that the child is likely to have need for in his

practical living to enable him to carry them on without hesitation and without making an undue number of mistakes.

The development of the arithmetical operations

The best order of teaching addition, subtraction, etc., is a matter of debate. Another question of procedure, the solution of which rests upon the analysis we make of the number operations, concerns the order of teaching the fundamental operations, — addition, subtraction, multiplication, and division. The traditional method is to teach addition, subtraction, multiplication, and division in the order named. There is some ground, however, for thinking the order should be reversed, and that we should begin with division and multiplication and then proceed to subtraction and addition. We may examine the grounds for these two forms of procedure and attempt to determine which one of them is the better adapted to introduce the child to the complexity of the number processes.

Multiplication and division involve the equality of the smaller groups. We may first contrast addition and subtraction taken together, with multiplication and division. Addition and subtraction differ from the other two processes in that in them we deal with unequal groups. When we separate a group into a number of smaller groups, in division, we obtain groups which are of equal size. In the same way, when we multiply, we take the same group a number of times

until we arrive at the larger group which results from this multiplication. This fact of the equality of the groups in multiplication and division would seem to make these processes simpler than those of addition and subtraction.

Addition and subtraction are probably simpler to the child. On the other hand, addition and subtraction may be carried on one step at a time, as when we have four and add one to it, or when we add two to four. In this case it is necessary to keep in mind the first number and the number which is added and the resulting number. When we multiply, we have to keep in mind the number of the smaller groups, the number of groups which are taken, and the final result. To the adult who is familiar with the result of such a combination it seems to be fully as simple an operation as is that of addition. But it is doubtful whether the child who is learning the meaning of multiplication and division in the first place can readily jump from the idea of a certain number of equal groups to their combined number. He, in all probability, has to proceed by taking one group at a time and calculating the result by the addition of each succeeding group to the preceding ones by a process of counting. The idea of the equality of the groups, then, is in reality a complicating element which makes the whole process more difficult. The same line of reasoning applies to division. The child probably has to approach the process by taking away one group at a time and counting the

remainder instead of by making a division into several groups all at once. It appears from observation that he begins multiplication or division by counting and that he arrives at the result first without using the idea of the equality of the groups. This procedure will be sufficient in addition and subtraction, but in order to understand multiplication or division he must have grasped the fact also that the groups are equal.

To take addition before subtraction seems to be in harmony with the child's habits. The further question may be asked whether we should take subtraction before or after addition, and division before or after multiplication. Here again the traditional method has been to begin with addition and multiplication and then go to subtraction and division, but there has been some disagreement with reference to this practice. In the case of addition and subtraction, the child clearly has the habit more firmly fixed of proceeding from a smaller to a larger number than a habit of proceeding from a larger to a smaller number. This is true in the case of counting, and it is illustrated by the fact that it is very much easier for the child to count forwards than backwards. So far as the manipulation of grouped objects is concerned, it is of course easy to take several objects from a larger number and determine how many there are left, but, on the other hand, it is just as easy to add a number of objects to a smaller group and count the number which results.

Division presents more possibility of difficulty than

multiplication. There is a more radical difficulty which attaches to the process of division as contrasted with multiplication. This difficulty is due to the fact that not all numbers can be factored and that numbers which can be factored have only a few factors. The child can be taught to divide first by using only the numbers which have factors and using only those factors which result in no remainder. Here again, however, there seems to be no greater difficulty for the child in putting together the series of equal groups and determining what the result is than in dividing a larger group into a number of smaller groups and determining the size and number of each smaller group. Since there are greater possibilities of difficulty in division, it seems to be the better plan to defer this process and begin with multiplication.

Multiplication and division introduce the principle of ratio. This description of multiplication and division shows that they involve the idea of ratio, or at least the idea of ratio may be developed from them. When the child recognizes the fact that a number — as, for example, six — may be divided into three groups of two units each, it is but another way of looking at the same fact to say that two is one third of six. That three twos are six, and two is one third of six, may be presented as different ways of stating the same fact. There is no need of making the matter mysterious or difficult by any discussion of the ratio idea in the simpler number processes, as is sometimes

done. It may be said that the idea of number involves the idea of ratio from the start. The idea of four is implicitly four times one, and the idea of the unit is implied in the idea of the number, but to discuss this matter with children is to attempt to call attention to a matter that is entirely taken for granted, and this raises a serious difficulty. The idea of ratio, then, comes in most naturally when the child deals first with multiplication and division.

Ratio may be represented by grouped objects or by measurement. This ratio idea may be very readily represented by the use of grouped objects. If objects are arranged in a regular fashion, — for example, if six objects are arranged in three groups of two each, — we have a very simple and easy mode of representation of the idea that two is one of three groups which make up six, or is one third of the group. The ratio idea may also be represented by measurement, and for the simpler ratios this is, perhaps, somewhat more direct and easily grasped. The traditional illustration of a pie which may be divided into three or four or more parts is apropos in this connection.

Fractions are merely symbols to express ratio. In explaining the method of representing fractions in number rotation, it is possible to make unnecessary difficulties for the child by going into too much minute detail. The first association may be merely between the term "one third" and the symbols of the one and the three expressed as a fraction. When we go further

to explain what the one and the three mean, it may be explained that the numerator refers to the number of groups which are represented in the fraction and that the denominator refers to the number of groups in the total number which is represented.

Operations with fractions are easily illustrated with grouped objects. In the operation with fractions, such as in addition of fractions, grouped objects are probably more readily applied than measurement. All that is necessary in this case is to represent both ratios as a part of total groups of the same number of objects. This is the concrete mode of representing the reduction of the fractions to a common denominator. An illustrative case is shown in the accompanying figure. In order to add three fourths and one sixth we may let both fractions be represented by the appropriate part of the larger group of twelve. In the one case this group of twelve is divided into four groups and in the other case into six. If we then take three of the first division and one of the second, we can determine what our result will be by merely counting the number of

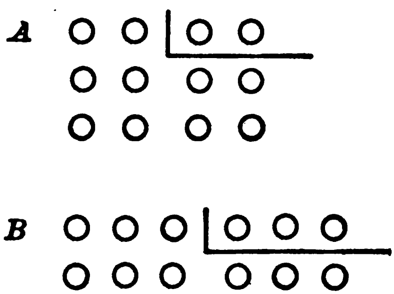

FIG. 10. ILLUSTRATION OF THE USE OF GROUPED OBJECTS IN NUMBER OPERATIONS

units in the various groups. Some such concrete method is undoubtedly desirable to give the child an understanding of the meaning of fractions and of the operation with them.

Percentage is a special case of fractions. Percentage is merely the extension of the principle of fractions to a special case and the use of a special means of representing these fractions. After the child has grasped the decimal form of grouping, as it exists in our number system, he is prepared to understand that percentage represents a case of fractions in which the denominator is ten or a multiple of ten. Expressed in concrete terms, this means that the whole group, of which the fractional part is represented in a percentage, is always made up of ten, one hundred, or another multiple of ten.

Decimal notation applies to fractional parts of a unit the principle we have already considered. In a similar fashion our mode of representation of percentage may be grasped as an extension of the mode of representing whole numbers. The child has already got a notion of the meaning of the places in writing a number, and understands that the digits in the first place represent single units, those in the next place to the left represent higher units each of which is a group of ten, and those of the next place a higher unit each of which is a group of one hundred, and so on. If we now extend the number system to the right of the units place, we have a series of representations of fractional parts of a unit.

In particular, if we suppose that the unit as represented in the units place be divided into ten parts, then the number in the next column to the right represents the number of these parts which are designated. The number in the next place represents a division in a like manner of each unit of the first decimal place into ten smaller units, and so on. Or if we start with the right end of any number, whether it include decimal places or not, each place to the left represents a unit which is ten times the unit represented by the next number to the right.

Decimal notation is easily illustrated by grouped objects. These various units of higher and lower orders, and their significance in the different places of the number notation, may be represented by using grouped objects, as has been suggested. If we use grouped objects to represent decimal fractions, we start out by making the units which are represented in the first or units place, not single objects, but a composite unit, or a group. The child has become familiar with a composite unit from the earlier familiarity which he has with the meaning of the tens, hundreds place, etc. If the first place, then, represents a unit made up of ten parts, it may be clearly seen that the next place to the right — that is, the first decimal place — will represent a number of these smaller units which are of the lower order.

The difficulty of grasping the number idea and its mode of representation should be separated in order

that the child may understand as much as is practicable. We have here brought home to us a fact which was referred to earlier, namely, that the child is confronted with two sources of difficulty in his study of number. He is required not only to understand the number relations themselves, but also the means by which they are represented by the number symbols. Very frequently the difficulty which he has is mainly with the symbols. This emphasizes the conclusion that the two difficulties should be separated to the extent that the child should grasp the idea first in its concrete form, and then the mode by which it is represented through the number symbols, making the connection between the two. When he starts with the number symbols they are meaningless to him, and he has no clue to the understanding of their complexities. The only course which is open to him, in this case, is to memorize the processes by rote memory and trust solely to his memory to guide him. Thus mathematics, instead of being a means of the development of thinking, becomes merely a blind, mechanical exercise of memory. This is unfortunate and is to be avoided as far as may be.

The mental processes in algebra

Algebra is a further development of arithmetic. Up to this point we have traced the development of number into more and more complex forms so far as it is included in the phase of number which we call "arith-

metic." In the common-school practice this has been marked off sharply from another form, which we call "algebra." In reality the two are not radically different, but algebra is merely an advance over arithmetic in the same direction. Algebra is fundamentally like arithmetic in that it represents relations of numbers to one another. It differs, it is true, as we shall see, but the difference is not such a complete one that it is necessary to separate the two into entirely different subjects of study. As a matter of fact, it is desirable to introduce some of the algebraic forms of procedure into the later stages of arithmetic.

Illustration of the arithmetical and algebraic methods of solving the same problem. We may first make clear the relation of algebra to arithmetic, as well as the difference between them, by an illustration; and then discuss the meaning of this difference more particularly on the basis of this concrete example. The following example may be worked either by an arithmetical form of procedure or by using an algebraic method. It is an example given by D. E. Smith in *The Teaching of Arithmetic*, page 74. The problem is: "If some goods are sold for $1,012.50 at a profit of 12½ per cent, what did they cost?"

The common method of solving this problem according to arithmetic is to let 100 per cent represent the cost and 100 per cent plus 12½ per cent, or 112½ per cent, represent the selling price. Then, dividing $1,012.50 by 1.12½, we have as a result $900, which

represents the cost. This form of procedure will give the answer, but it is difficult to make the child understand why it is taken. Why do we let 100 per cent represent the cost, and why do we divide $1,012.50 by 1.12½? It is difficult to make the child recognize that this is more than an arbitrary procedure.

The other procedure is to let some letter, such as *c*, represent the cost. This is a natural letter to take, since it is the first letter of the word. Then the selling price is *c* plus 12½ per cent of *c*, since the profit was 12½ per cent reckoned upon the cost. The selling price then is 1.12½ *c*. This in terms of the equation becomes

$$1.12\tfrac{1}{2}\, c = \$1{,}012.50.$$

The equation may then be solved by dividing both terms by 1.12½, when we get as a result —

$$c = \$900.00.$$

By using illustrations from concrete numbers the procedure of dividing both terms of the equation by the same number may be justified without going into a full explanation of it at this time.

The symbols of algebra are more abstract than those of arithmetic, since they do not represent definite numbers except as they are defined by other elements in a problem. This method of solving a problem illustrates the essential advance of algebra beyond arithmetic. In arithmetic, each symbol, such as the symbol 5, represents a definite number. Each number may be illustrated by reference to various kinds of concrete objects. It is abstract in so far as it may be

applied to any sort of objects which we choose to regard as units. In the algebraic symbol, however, we have a means of designating any number whatever. The symbol *c* which was used in this problem represents a number which is not known until the problem is solved. The advantage of this is that we may use this symbol as a term in our number manipulation without knowing what actual number it represents. The symbol represents a definite number so long as the other terms used in the problem are defined, but if we take an algebraic formula in which all of the symbols are of the indefinite algebraic sort, then the formula will hold true whatever definite numbers are substituted for the algebraic symbols. If certain of them have numbers substituted for them, this restricts the possibilities for the others. The only requirement then is that the same symbol be represented by the same number throughout the solution of the problem.

Arithmetical symbols may represent indefinite quantities, but in a roundabout way. It will be well to clear up a question which may occur with reference to the statement which has just been made. One may ask how it is that, if the arithmetical symbols — that is, the digits or numbers made up of them — can represent only specific numbers, and the algebraic symbols alone can enable one to solve problems in a different way because they represent unknown numbers, or may represent a variety of definite numbers, the above problem can be solved by arithmetical means. In

order to answer this question, we must qualify the statement that the arithmetical symbols represent only definite numbers. We have already seen in the previous discussion of arithmetic that a combination of numbers in the fraction, representing a ratio, may represent a more abstract relationship than is represented by a single number. Three quarters represents nine when the total group which is thought of is twelve, but it represents twenty-four when the total group is thirty-two, and seventy-five when the total group is one hundred. The use of percentage as a form of application of ratio enables one to use the symbol which may have a variety of definite numerical quantities. In the case before us the cost was represented by 100 per cent, that is, by the ratio $\frac{100}{100}$ and the profit by $12\frac{1}{2}$ per cent, or $\frac{12\frac{1}{2}}{100}$. These ratios, then, are of the same nature as the algebraic symbol *c*. The difference is that the unknown quantity may be more conveniently represented by a symbol which does not imply any definite number than by a ratio which is itself made up of two definite numbers. It is more easily grasped as a symbol which represents an unknown quantity than is the numerical ratio. It is true that both methods may be pursued in such a case as this, but in the more complicated forms of operation, the use of symbols which evidently represent an unknown quantity is easier and better understood.

Manipulation of the equation is a central element in algebraic procedure. The above problem illustrates

also another fundamental fact in regard to algebraic processes. After the algebraic symbol had been introduced, the problem was stated in the form of an equality between two quantities, one of which contained the symbol for the unknown quantity. This statement of an equality is the equation, and the equation, with the rules and the principles for manipulating it, is one of the most fundamental elements in the algebraic form of procedure.

The equation merely states the result in arithmetic, but is used to obtain the result in algebra. The equation is *implied* from the very beginning of the study of arithmetic. When the child realizes that three groups of four each are equivalent to a group of twelve, he is recognizing an equality between two forms of grouping, and may state the equality in the form of an equation. In arithmetic, however, the operations are all performed on one side of the equation and the equation is used merely to state the result of these operations. Take a more complicated form of multiplication, such as 24 times 30. In this case the multiplication of the two numbers is carried on as a series of processes and then the result is stated in the form, $24 \times 30 = 720$. The equation only expresses the result of the multiplication which has been carried on with the terms on one side of the equation. In algebra, on the other hand, the problem is stated first in the form of an equation, and the solution is reached by manipulating the terms or the symbols not merely on

one side, but on both sides, or by adding, subtracting, or performing other operations alike with both sides.

An illustration of the use of the equation in algebra. Here again we may make the matter clearer by illustration. Take as a problem one of Sam Lloyd's puzzles. The following problem serves the purpose: "At a certain time between seven and eight o'clock, the two hands of a clock are at equal distance from the mid-point of the dial at the bottom, that is, from the point that represents the half-hour. At what time are the hands in such a position?"

The conditions of the problem must first be put into algebraic symbols. It is obvious that we have here a complicated set of conditions which are difficult to express or to work out by using only the definite quantitative symbols of arithmetic. Suppose, then, that we use a symbol to represent one of the unknown quantities, which, if it were known, would enable us to answer the problem. Let x, this unknown symbol, represent the number of minutes past the hour which the minute-hand has progressed, or in more general terms the number of minutes past the hour which is represented by each of the two hands. We may now make up an equation from these assumptions. The position of the minute-hand to the right of the bottom or the mid-point of the dial will be represented by the difference between thirty minutes and the minutes past the hour which the hand indicates; that is, the position will be represented by $30 - x$. This distance

is equal, according to the conditions of the problem, to the distance from the bottom mid-point to the hour-hand. This distance we can also represent by means of our symbols. Since the hour-hand progresses through a distance representing five minutes on the dial, in the same time that the minute-hand progresses throughout a distance representing sixty minutes, and since they progress at a proportional rate of speed, the distance which the hour-hand has traveled from the seven o'clock point toward the eight o'clock point will have the same ratio to five minutes that the distance the minute-hand has to travel has to sixty minutes. Since the distance the minute-hand has traveled is represented by x, the distance which the hour-hand has traveled is represented by

$$\frac{1}{12} \text{ of } x, \text{ or } \frac{x}{12}.$$

The position of the hour-hand beyond the bottom mid-point, then, is $5+\frac{x}{12}$, since it started at the beginning of the hour at a point five minutes beyond the thirty-minute point. According to the conditions of our problem, these distances are equal, and we may state the problem in mathematical terms as follows: —

$$30-x=5+\frac{x}{12}.$$

The equation must then be solved. So far the problem has been merely to put into algebraic terms the statement of the question. The rest of the solution of the problem consists in so manipulating the equation

which we have stated that the value of x will be found. The operations are represented according to the familiar manner, as follows: Transposing the 5 and the x so that the numerical quantities are on the same side and the values of x are together, we get as a result

$$\frac{13}{12}x = 25.$$

Then $$x = \frac{12}{13} \times 25, \text{ or } 23\tfrac{1}{13}.$$

If we apply this value of x to our problem, we find that the distance which the minute-hand is from the thirty-minute point is $30 - 23\frac{1}{13}$, or $6\frac{12}{13}$, and that the hour-hand is distant from this same point $\frac{23\frac{1}{13}}{12} + 5$, or the same number, $6\frac{12}{13}$. The process which is involved here, besides setting the problem into algebraic terms, is the solution of the problem by manipulating the equation. This latter part of the solution necessitates learning the principles which govern what can be done with an equation. In general, these may be summed up by saying that what is done to one side of the equation must also be done to the other. This principle, of course, works out into many detailed forms of procedure, but it is one of the fundamental principles of algebraic work.

Algebraic expressions may be made concrete by substituting numerical quantities for the symbols. When an algebraic operation, which is represented by the algebraic symbols and their manipulations, becomes obscure, the matter may be cleared up and the

terms made more concrete by the substitution of numerical quantities for the symbols. This is, in principle, the same sort of process that is carried on when we substitute concrete objects for numbers in early number work, — as when the child uses splints or marbles, or measures areas or lengths, in order to illustrate or make concrete the meaning of numbers and of their combinations. In the same way we approach the concrete in algebra when we substitute definite numbers for the algebraic symbols.

Negative numbers require an intricate mode of representation. Some of the notions which are employed in algebra may be represented concretely, though by somewhat intricate devices. One of these is the idea of negative numbers. If we substitute numerical quantities for negative algebraic symbols, we cannot manipulate them in the same way in which we do positive numerical quantities, and we cannot illustrate their meanings in the same simple way by substituting concrete objects. In order to grasp the meaning of negative numbers and of the operations which are made with them, it is necessary to use somewhat more complex spatial relations and to illustrate by a combination of these relations and of movements through space.

Addition and subtraction may be illustrated by distances on a line from a middle point. To represent positive and negative quantities for the purpose of addition and subtraction is fairly simple. All we have

to do is to use a straight line with a dividing point in the middle, and represent positive quantities by distances toward the right and negative numbers by distances toward the left. If we have to add a positive and a positive number, we merely add the corresponding distances in the same direction and we arrive at a result which is equal to the sum of the two numbers. The addition of a positive and a negative number is represented by a distance toward the right and then a

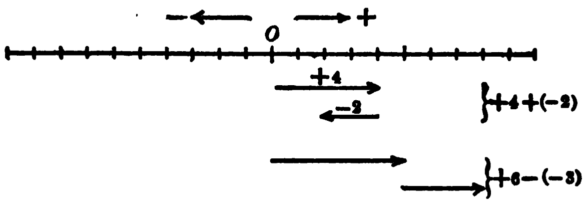

FIG. 11. CONCRETE REPRESENTATION OF OPERATIONS WITH NEGATIVE NUMBERS

distance from this point to the left, each corresponding to the size of the number. (See Figure 11.) The operation then is performed by subtracting the negative number from the positive, or the positive from the negative, and affixing the sign of the larger number. The subtraction of one number from another can be represented by movements in the direction opposite from that indicated by the sign of each number. Thus, if we subtract a negative from a positive number, it is the equivalent of adding two positive numbers.

In the case of multiplication we must resort to a still

more complex form of concrete representation. Multiplication of negative numbers may be illustrated by the accompanying Figure 12, adapted from Myers's *First Year Mathematics.* The line AB may be taken to represent a bar which turns about a fulcrum in the center. The distances along this bar toward the right

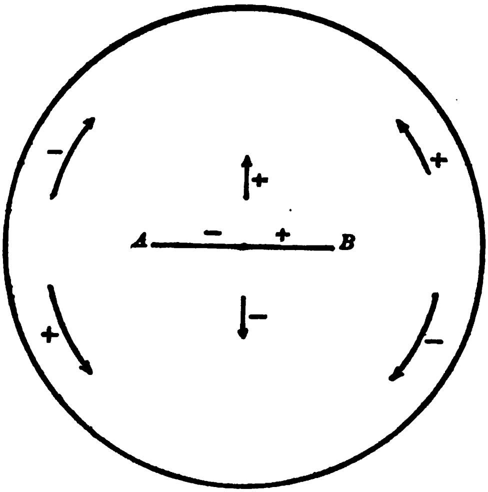

FIG. 12. CONCRETE REPRESENTATION OF OPERATIONS WITH NEGATIVE NUMBERS

are represented by positive, and along the left by negative, numbers, as in the case of addition and subtraction of negative numbers. A force acting upward upon this bar represents a positive number and a force acting downward, a negative number. The two numbers which are to be multiplied are represented, one by a position upon this bar to the right or the left of

this fulcrum, and the second by a force acting upon it. The result is represented by the turning of the bar, and when this is in a direction opposite to the movements of the hands of the clock, it is designated as positive, and when it is in the direction the same as the movement of the hands of the clock, it is designated as negative number. The sign of the product may then always be found by representing one of the numbers which is to be multiplied by a position on the bar corresponding to its sign, the other number acting in the direction corresponding to its sign, the sign of the result corresponding to the movement produced on the bar. Thus a positive number multiplied by a positive, represented by an upward force acting upon the right side, always results in a positive result or multiplicand. Similarly, a negative number multiplied by a negative results in the same way. On the other hand, a negative number multiplied by a positive always results in a movement in the negative direction.

These illustrations make clear the abstractness and complexity of algebra. It is evident that we are here getting into an abstract matter, since we represent the mathematical relationship by such a combination of directions, distances, movements, and resultant movements. These may serve as examples of some of the more important of the ideas which are developed in algebra. They go beyond the arithmetical processes; and in virtue of this fact, they make it possible

to handle number relations in a way that is not possible in using the more definite and less manageable number symbols. Algebra is both a more complex and a more valuable tool than is arithmetic and requires for its mastery a higher degree of abstractness and a higher type of intellectual process.

The development of geometry from measurement

In the discussion of number and its development in arithmetic and algebra we saw that the various relations of numbers may be illustrated by means of measurements of lengths, areas, or volumes; and that these may be used as illustrations of the relationships between numbers and the various number operations, such as addition, multiplication, etc. From that point we have traced the lines along which the development of number has been specialized. We have met more and more complicated ways in which numbers may be dealt with in the various operations of arithmetic, and have seen how, by the use of more abstract symbols, the more general operations of algebra are possible. The early form of measurement which is used to illustrate number is developed in another direction by giving particular attention to the relations of forms and spaces and distances, and to the general principles which govern them. The development of the principles governing the spatial relations is called "geometry." The relations of geometry to measurement may be seen from the word itself. In its origin, the

word meant the measurement of the earth, and, in conformity to this meaning, geometry was developed out of measurement of land for practical purposes.

In the school and in the history of the branch there is a distinction between measurement and formal geometry. In the school curriculum there has been a sharp division between the early measurement which the child carries on and which may be used to represent the number processes and the later refinement of these same studies of spatial relations as it appears in the form of Euclidian geometry. The distance or gap which separates the early or crude measurements from the formal study of the principles of the relations of space is one which has resulted from the fact that our geometry is inherited from the Greek philosopher Euclid. Geoemtry in the English schools has always been called by the name of Euclid, who first systematized the principles of the subject. Many of these principles were known in a concrete way before Euclid. Many of the general propositions of Euclid were discovered by practical experimentation, and were practically applied, before Euclid worked out their proof and formulated his principles. The Egyptians understood the relations between the diameter and the circumference of a circle and between the hypothenuse and the legs of a right-angled triangle, but they had not worked out the formal demonstration of these relationships.

Accurate measurement may give the child an early

knowledge of many geometrical principles. These historical facts suggest that many of the facts regarding space may readily be learned by the child before he undertakes the study of their rigid proof. The gap between the early crude measurements, which are connected with the first appreciation of numbers, and the formal study of geometry might then very well be filled in by a somewhat more exact use of measurement through which the general principles of the spatial relation could be experimentally discovered. This would prepare the child who is to go on to the formal study of geometry by giving him a better concrete basis for his more abstract study of principles, and it would furnish to the child who is not to go on the direct knowledge of spatial relations which would be of value to him.

Many geometrical generalizations may be worked out concretely before the child can understand their demonstration. That geometry has been placed later than algebra in the high-school curriculum is due to the fact that the extremely abstract form of geometry, or the study of space, has been used to the exclusion of the more concrete study of the same facts. If geometry is studied through measurement, it may be considerably more concrete than algebra or even some of the topics of arithmetic. Thus, a pupil may gain a practical appreciation of the fact that the area of a rectangle is measured by the product of the length of the two adjacent sides, or even of the fact that the

square on the hypothenuse of a right-angled triangle is equal to the sum of the squares on the two legs, without going through any rigid demonstration of these facts. Thus the child will come to recognize certain geometrical facts in the same way that he learns to make other generalizations. This procedure consists in recognizing that a fact holds true in a number of cases, and in the conclusion that it is true in general. Take the first illustration which was used. If the child has measured a large number of rectangles, and finds that the squares which are made by drawing lines at unit distances to connect the opposite sides are equal in number to the product of the unit distances on the adjacent sides, he will arrive at a generalization that this is always true of rectangles. There is no reason why he might not properly make this generalization before he has learned its rigid proof. We do not wait to teach the child generalizations in natural science, until he is able to appreciate their scientific demonstration.

The Pythagorean Theorem may be illustrated concretely. Take two illustrations of the appreciation of spatial relations of a fairly complex sort through the method of measurement, or of direct apprehension, rather than through demonstration. The Pythagorean Theorem serves as a good example. In Figure 13 we have a special case of the theorem. It may be illustrated for the child by having him construct a right-angled triangle with one leg four units long and the

other three units long, and then having him construct the squares upon the three sides. He will find that the hypothenuse measures five units long, and then, if he divides each of the squares into smaller squares of

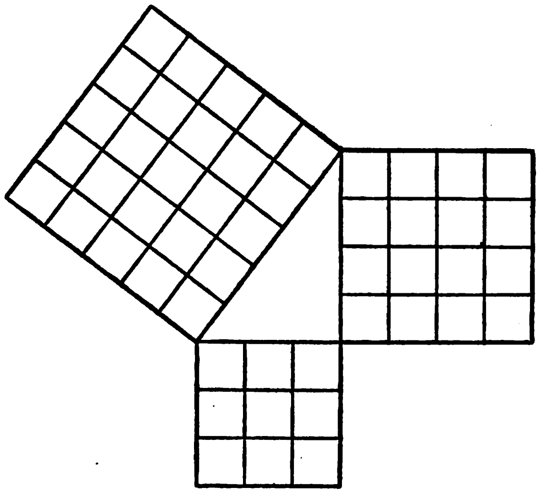

FIG. 13. ILLUSTRATION OF THE SOLUTION OF THE PYTHAGOREAN THEOREM BY MEASUREMENT

unit length on each side, he will find that the two legs are measured by squares of sixteen and nine units, and the hypothenuse by a square of twenty-five units. Thus he can see that in this particular case the square upon the hypothenuse is equal to the squares upon the two legs.

A more general mode of representation may also be used. A similar relation for right-angled triangles in

general may be well illustrated by Figure 14, which is a little more complex and suited to a more advanced stage. *ABC* is a right-angled triangle and *ABKD* is a square constructed on the hypothenuse. *ACEF* and *BCHG* are squares constructed on the two legs. Let the child connect *D* and *F*, extend *CE* and drop the perpendiculars *KJ* and *KI*. He may then determine by measurement or by superposition that *KJEI* and *BCHG* are equal. Therefore, the irregular figure *KJFACI* represents the squares on the two legs. If this figure can be shown to be equivalent to *ABKD*, the general truth of the theorem is evident. This can be done by superimposing *ABC* on *AFD* and *BIK* on *KDJ*.

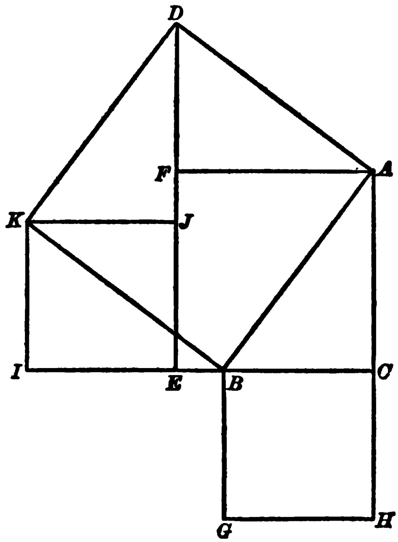

FIG. 14. ILLUSTRATION OF THE SOLUTION OF THE PYTHAGOREAN THEOREM BY MEASUREMENT

This generalization is to be distinguished from rigid proof. In the stage that we are supposing, the child is not expected to prove the equivalence of the

two figures. He may, however, see that they are equivalent by measuring the various parts as indicated, provided that the figure has been constructed with sufficient care so that it is accurate. The proof of such a conclusion as this requires much more than the demonstration that it is true in one particular figure. The child may recognize its truth and may generalize upon this so as to conclude that it is true for all figures of certain specifications; but to prove that it *must* be true for every figure which meets these specifications is a different matter.

Rigid proof bases conclusions on assumed axioms or on previously proved conclusions. An advance to the geometry of proof depends on the appreciation of logical requirements. The final stage of development in geometry, then, is not merely the recognition that a certain relation holds true, but rather the appreciation of the answer to the question why this relation must be true. The very obviousness of the fact that a fact is true is frequently the greatest source of difficulty to the pupil in determining why it must be true. To raise a question that the shortest distance from a point to a straight line is a perpendicular seems a foolish thing to do, and this is true also of many other geometrical theorems. The pupil is not ready for this demonstrational type of geometry until he can appreciate the requirements of a logical series of steps, each of which depends upon some assumption which is either an axiom or a previously demonstrated theorem.

Geometry proceeds from concrete to abstract as do arithmetic and algebra. When the form of proof has been gone through in this manner, it becomes a general or abstract affair which does not rest merely upon the appreciation of a relation in a certain concrete figure. In geometry, as in arithmetic and algebra, we begin with the understanding of certain concrete relations, or relations among concrete objects, and then gradually progress until the relations can be understood abstractly. When this stage is reached, the pupil uses a concrete figure only as an illustration or a mode of representing the conditions of the problem to his mind. The proof of this problem rests upon the appreciation of the dependence of the theorem upon a series of previously demonstrated theorems.

An illustration of the discovery of a geometrical proof. The following illustration may serve to show how one proceeds in geometrical proof. Suppose that we set out to prove that a straight line which is perpendicular to each of two intersecting straight lines at their point of intersection is perpendicular to the plane *P* of these lines. This means that the line must be proved to be perpendicular to any other straight line whatever of the plane *P* which intersects at the same point. In Figure 15, assume the line *AO* perpendicular to *OB* and *OC* in the plane *P*. To prove that the line *AO* is perpendicular to any other straight line in the plane drawn through the point *O*, as *OD*.

The conditions of the problem are first clearly real-

ized. The pupil first attempts to get clearly into mind what the conditions of the problem are, and then to cast about for methods of proving, on the basis of the conditions which are known, that which is to be proved. We know that AOC and AOB are right angles, and we wish to prove that AOD is a right angle. One method with which the student has already become familiar is the use of triangles. One may construct a number of triangles by drawing the lines AB, AC, and CB, then another triangle may be constructed, or a series of triangles, by connecting A with the point of intersection of OD and BC at D by the line AD.

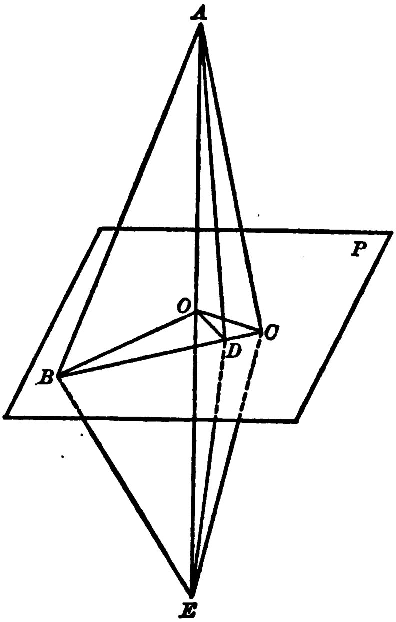

Fig. 15

The student follows up promising clues. This, however, does not seem to lead directly to the proof. Another form of procedure which is commonly used is to extend a line, such as AO, through the plane to an

equal distance on the other side, represented at *E*. By connecting *B*, *C* and *D* with *E*, we have another series of triangles. It is obvious that if the triangle *AOD* can be proved to be equal to *EOD*, the angles *AOD* and *EOD* will be proved to be right angles. A number of triangles are easily proved to be equal, as *AOC* and *EOC*, and *AOB* and *EOB*. This suggests that the triangles *ABC* and *EBC* may be shown to be equal, because the three sides are equal, and that by this means the triangles *ADC* and *EDC* can be proved to be equal. As a consequence *AD* and *ED* are equal. This makes it possible to prove the equality of *AOD* and *EOD*, and therefore of the angles *AOD* and *EOD*, and proves that they are right angles.

The process of problem-solving is usually more random. The student in solving this problem would commonly not have arrived at the steps in such a direct manner. A great many more useless ideas would have been considered and rejected before the series of steps which led to the final solution was finally hit upon. The degree of directness with which the student proceeds to the conclusion depends somewhat on the amount of his previous experience in solving such problems, and upon his familiarity with the various possible methods of approach.

Problem-solving

Problems are solved by analyzing conditions presented and following up clues. The procedure which is taken in solving such problems illustrates the general method of attacking such problems, and further illustrates the method of problem-solving or of reasoning in general. We may very briefly summarize the characteristics of such procedure. In the first place, one starts out with a problem, which may be in the form of a certain conclusion which is to be tested. With this problem in mind, the conditions are analyzed, or broken up, in order that one may get a clear realization of the various elements of the problem. One then casts about in his mind for various clues to the way in which the solution may be approached. In this he is aided by methods which have been found to be suitable in similar cases in the past. The methods which are suggested are then tried out until one is found which leads to the appropriate series of steps and finally to the conclusion. These steps must be so connected that each following one depends upon the one preceding.

Solving a problem is radically different from following the statement of a completed proof. The means by which the steps in a solution are stated after the solution has been reached must not be confused with the steps through which one goes in reaching the conclusion. The identification of these two is responsible

for the fact that many students of geometry never learn to attack the problems themselves. The demonstrations are stated by building up the proof in a logical manner, step by step, by beginning with known facts and proceeding in an orderly way to the conclusion. The proof is reached by the student, however, by beginning with the conclusion which is to be proved and working back to the facts which may serve as the basis for such a proof. In a measure the two processes begin at opposite ends, and one should distinguish them in mind as well as in practice.

Problem-solving appears most prominently in geometry. This consideration of problem-solving in geometry leads us to the question of problem-solving in number work in general. Problem-solving is a somewhat more intimate part of the work in geometry than of arithmetic and algebra, because of the fact that each new theorem is a new problem in itself and is a step in a whole series of problems which are closely knit together. In the case of arithmetic and algebra, the particular problems which the child is set to solve are grouped about certain general types of solutions. The pupil is led to apply a certain method of solution to a large number of concrete situations. In this process he is not developing a new proof or the proof of a new fact, but is applying to a variety of situations a method which he has learned to use.

The grasp of the statement of the problem is important in arithmetic and algebra. Although there is

this difference in problem-solving in geometry and in the other branches of mathematics, yet the general features of the mental process are the same in the different fields. In the case of arithmetic and algebra, however, we meet with a phase of the matter which is prominent in them and which is one of the important conditions which govern the ability to solve problems. It has been found by experiment that the difficulty in solving an arithmetic or an algebra problem depends in a large measure upon the ability of the pupil to understand what the statement of the problem means. Courtis found as reported in Bulletin number 2 of the *Courtis Standard Tests*, August, 1913, that the same problem could be stated in a variety of ways, and that the success of the pupil in solving a problem varied remarkably with the variation in the manner of stating it. There is a distinction between the appreciation of a mathematical relationship, or a method of manipulating mathematical symbols to obtain certain mathematical results, and the ability to apply these mathematical processes to a practical concrete problem. One may understand the mathematics itself very well and yet not know what process to choose to solve a special concrete problem.

Understanding a problem usually involves a clear grasp of concrete objects and relationships. The first step in solving a mathematical problem is the understanding of what it means in its concrete terms. The failure to reach this understanding is seen sometimes

not merely in problems in arithmetic and in algebra, but also in geometry. The student of geometry, particularly of solid geometry, sometimes fails to realize that the lines which are used to represent the figures represent spatial relations. Until a student can get a clear grasp of the sort of figures which are intended, he is not in a position to understand in any measure the problem or its demonstration. In the case of problems in arithmetic or in algebra, the first requirement is that the pupil shall be able to picture to himself just what the meaning of the problem is. This very frequently means the ability to form an image of certain concrete objects. Whether such an image is absolutely essential or not, it is very likely that it is in all cases beneficial, and that the clearness of grasp which is attained by the pupil will depend upon the clearness with which he can picture to himself the concrete setting of the problem. Take the familiar problem concerning the rate with which a tank would be filled if water flowed in through a pipe of a certain size and out through a pipe of another size. The ability to picture the conditions of this problem will make it easy for the pupil to know what sort of formula to use in its solution, or at least will prepare the ground for the knowledge of the mathematical form of procedure which should be used.

The significance of the mathematical procedure must be clearly understood. The next step is obviously to turn the problem into mathematical terms or to know what form of mathematical solution ought to be

used in a particular case. This means that the pupil should grasp not merely the meaning of the problem as it is stated, but also the meaning of the various forms of solution which he has learned. It frequently happens that pupils are able to solve problems which are classified under the form of procedure which is appropriate for them, but are not able to solve similar problems when they are arranged in miscellaneous order so that there is nothing to indicate which mathematical formula is to be used. When a pupil shows a great difference between his ability to solve the classified and unclassified problems, the probability is that the problems have not been real to him, either because he did not appreciate their meaning, or because he did not understand the meaning of the mathematical formulæ used. In either case his solution of the problems is merely a matter of mechanical juggling of terms without an understanding of their use and is of no value to him from the standpoint of intellectual development.

Much practice in discovering the general mode of solution of problems is desirable. It is well for the pupil to have a large amount of practice in the understanding of problems in their concrete statements and in determining what form of solution should be used for them. This can be done by arranging series of statements of problems and requiring the pupil not to work them out in detail, but to put their meaning in other words and to state how one should proceed in

general to solve them. By this method the pupil will learn to distinguish and understand the general features of the various methods of procedure because he is not confused with a mass of details.

Number illustrates the higher thought processes. It has become evident, from this whole discussion of number and mathematics in general, how mathematics constitutes a form of thought which is not a direct reaction to the concrete things of the physical world, but rather consists in the development of complex systems of thought processes which enable one to deal with the concrete processes in an indirect manner. One first selects from the multifarious attributes of things the number aspect; and then develops forms of procedure with numbers wholly or largely abstracted from the concrete objects themselves; and finally applies the results of these operations to the concrete things. Instead of constituting an immediate reaction to the physical object, such a response is made up of a series of thought processes which are carried on for a time with little reference to the objective world. The next chapter will be a further illustration of this higher form of response.

REFERENCES

1. S. A. Courtis: "Standard Scores in Arithmetic." *Elementary School Teacher* (1911), vol. 12, pp. 127–37.
2. Frank N. Freeman: "Grouped Objects as a Concrete Basis for the Number Idea." *Elementary School Teacher* (1912), vol. 12, pp. 306–14.
3. C. H. Judd: *Genetic Psychology for Teachers*, chap. 9. Appleton, 1903.

4. J. A. McClellan and J. Dewey: *Psychology of Number*. Appleton, 1895.
5. W. S. Monroe: "Warren Colburn on the Teaching of Arithmetic." *Elementary School Teacher* (1912), vol. 12, pp. 463-80.
6. D. E. Smith: *The Teaching of Arithmetic*. Teachers College, Columbia University, 1909.
7. H. Spencer: *Education*, chap. 2. Appleton, 1861.
8. J. W. A. Young: *The Teaching of Mathematics*. Longmans, 1907.

CHAPTER X

NATURAL SCIENCE: GENERALIZATION UPON EXPERIENCE

Summary. We have seen how the child makes various kinds of responses to the experiences which are furnished him by the stimuli from the external world. He has to begin with a variety of native responses called "instincts." He adds to these or builds upon them a great variety of responses which are not instinctive. One type of response is the response of movement to stimulus, and we call the mode of learning by which these responses are formed "sensori-motor learning." In other cases it becomes necessary to organize the stimulus itself so that one comes to recognize it as having a meaning and becomes able to distinguish the meaning among the different objects which he finds it necessary to respond to. Not only does the child learn to respond by giving interpretation to objects, but he is able to retain impressions made upon him so that he may recall them at some future time and take them into account in making his responses. He is also able to present to himself, in his imagination, events or objects which are distant in time or space. The child is also capable of responding to certain abstract features of the surrounding world

and can build up systems of thought by considering these elements. This abstraction is illustrated by number.

Natural science deals with generalizations. In the study of natural science we find still another mode of dealing with experience. This consists of developing ideas or modes of response which are equally applicable to a variety of objects or experiences. Take an illustration from the very early experience of the child. In his early dealings with the physical objects about him the child has not yet learned the common laws which govern them. He has to learn by experience such a fundamental fact as that an object when released falls to the ground. After he has seen this happen with one object after another, he learns how to prevent objects from falling and acquires the habit of expecting to see objects fall when they are released. He appears to pass through a period when he becomes conscious of this general mode of action of things, and delights in dropping objects partly for the pleasure in experiencing the fulfillment of his expectations. Such an idea, which applies equally to a whole class of objects, we call a "generalization."[1]

A child's fear of a balloon is explained by the fact that it violates his expectation. A good illustration of this expectation in the mind of the child, that objects

[1] A generalization is an idea which is the outgrowth of a mode of response which is found applicable to a whole class of objects. It is usually embodied in the statement of a general principle.

will fall when released, may be seen in the fear which a child exhibited when he saw a toy balloon, which did not act in the expected way. Here is an object, which according to all previous experiences ought to fall when it is released, but instead of that it rises in the air. The child who was the subject of this observation was very much frightened by this experience, and it required some time for him to become accustomed to the unfamiliar action of the balloon and to enjoy playing with it. The child acquires an expectation that things will act according to certain regular modes of action or laws. He finds that in the world about him certain regular kinds of action can be expected. In the early stages he develops these expectations unconsciously, but this forms the basis for the development of more conscious generalizations later on.

Concept formation

The child forms concepts, or ideas of classes of objects; and general principles, or ideas of the characteristics or modes of action of these classes. The child's generalization upon his experience proceeds in two ways. In the first place, he learns to distinguish those classes or groups of things which have a certain common mode of action or which have something similar, as a result of which one is led to put them into a class. The second procedure is to examine in more detail the characteristics or modes of action of objects which have been classed together. To illustrate these

two processes, the child may exhibit an interest in distinguishing between cats and dogs, horses and cows, etc., and in learning to recognize the members of the different groups; or he may become more interested in learning about the habits or characteristics of the cat or the dog or the horse. For example, he learns to distinguish between cats and dogs by certain marks of difference, and by further experience to learn more about the particular habits of cats — that they meow and scratch, and of dogs — that they bark and bite. We may distinguish these two by naming the first "concept formation," or the formation of general ideas; and the second, the formation of general principles or laws. It is evident from the illustration that there is not a sharp distinction between the two processes.

The child early classifies objects according to their use. The earliest form of a child's scientific development consists in learning to class objects together which have common characteristics. The basis on which a child first learns to group objects, and to regard them as belonging to a common class, is their use. If a child below nine years of age is asked to tell the meaning of common objects with which he is familiar, he gives the definition in terms of their use. He will say that a fork is to eat with, a chair to sit upon, a table is to set things on, a horse to pull a carriage, and so on. What a thing can be used for and what it does is the characteristic which attracts the child and which

leads him to put an object into one class rather than into another.

The development of concepts is hastened by the use of names. A child begins to classify things roughly before he can talk. The infant early distinguishes between things as good to eat or good to play with, and distinguishes between his own family and others, and between those within his own family. The use of names greatly facilitates this form of development and leads to a finer distinction between things. When a child finds a name given to an object, this immediately directs his attention to that feature of the object which it has in common with other things which are given the same name. He would probably be much slower in making distinctions and comparisons if he were left merely to his own observation in the matter.

The mistaken application of names illustrates how they operate. The kind of mental process to which the child is led by the use of names is illustrated in cases where he makes a mistake in the application of a name. It is related by Preyer that a little boy had been accustomed to hear his grandmother given the familiar name. There was no means of knowing what he thought it was which led this name to be applied to his grandmother, until one day he saw another old lady who also wore a cap. He thereupon called her "grandmother" also, thinking apparently that a grandmother was a person who wore a cap. This illustrates the fact that he had, without thinking particu-

larly about it, selected a certain feature to which he thought the name applied, and when he saw another who possessed the same characteristic, he extended the name to her also.

Mistakes emphasize the importance of selecting the essential feature as a basis for classifying objects together. Another illustration shows how a child may group a great many diverse things together by selecting in each case some feature which is common to two things and thus finally reaching in his chain of thinking an object which is widely different from the one with which he started. A little boy saw a duck upon a pond and heard the word "quack" applied to the duck. He did not clearly separate in his mind the duck and the water and so applied the name to both. Later he used "quack" to refer to water or even to any liquid. Further, he applied the name to other birds than the duck, then to an image of an eagle upon a coin, and finally to coins themselves. We see how far this process of association may carry the child from the original object to which the name was given. These cases of accidental or erroneous association emphasize the opportunity and need for the direction of the child's thinking in the formation of his concepts or general ideas. They bring out also the fact that a concept is built upon the analysis of an object so as to determine what the essential feature is. It is necessary, in order that the child may have the correct notion, that he should select the essential feature and not some acci-

dental feature upon which to base his classification of the object. This thought process in selecting the correct feature is of considerable value, and names, therefore, may form a stimulus to valuable thinking on the part of the child.

The development of generalization

The child's early explanations are superficial. It may thus be seen that the process of grouping things together goes parallel with the process of familiarity with the characteristics of the things themselves. The more one knows about individual objects, the better he is able to tell to what class they belong, and to divide them up into finer and finer groups. In the beginning the child shows a disposition to follow the practice of primitive man and to depend upon a superficial similarity in putting things into the same class. When he meets with new objects, he shows a disposition to think of them after the analogy of things which he already knows. In this he is not unlike mankind in general, but as he gains in intellectual growth he is able to distinguish the characteristics of new things which make them fundamentally different from objects which he has met before. This tendency to interpret new things after the analogy of familiar things is shown in the story of the Indians, who, when they first saw a steam engine, were greatly puzzled until they explained the running of the machine by saying that there were horses inside. They were oblivious to

the fact that the running of a horse is as mysterious a thing as the working of the steam engine, for since they were familiar with the horse's activity, it did not present this difficulty to them. The difficulty to them lay in the unfamiliarity of the new thing rather than in an appreciation of the problem which really existed in explaining it.

The child is ready first to learn the more obvious and external facts about things. The child's early scientific procedure is after the same manner. It consists in gaining familiarity with the mode of action or with the characteristics of a great variety of things, without going very deeply into their explanation. Animals and plants are particularly suitable for the child's early study, both because of his native interest in them and in their habits, and because the familiarity which he gains with them will serve as a basis for the later, more penetrating study which he will have to make. He is very much interested in learning about such facts as the habits of animals. He may be told of the nest-building of birds, the differences in the kinds of nests built by different birds, the way in which the beavers build their homes, the different modes of gathering food and caring for the young, and so on. These facts should not merely be taught to the child through verbal instruction, but he should also be led in the early period to begin the habit of observation. It is surprising what a fund of information a child may gather if he once becomes interested in such a pursuit.

In a similar manner the child may learn about the habits of plants; the different kinds of seeds which are produced by different plants; the ways in which seeds are carried from one place to another, or are given such form that they will readily spread; the other means of growth of plants, as from bulbs, seeds, cuttings, etc. He may learn also of the care which is necessary for the growth of plants as well as of animals.

Care for plants and animals stimulates the child's interest and gives him moral training. Subjects of study of this sort are particularly appropriate for another reason. They give the child an opportunity, if conducted in the proper manner, of doing something with things, as well as learning about them. The child may find out about animals and plants by actually caring for them. The care of a pet and the cultivation of a garden plot greatly stimulate his interest and lead him to learn a variety of things in a way in which he will not forget. Incidentally the child learns many lessons which are not directly related to scientific training, such as the lesson of thoughtfulness and sympathy in caring for animals, and the lessons of industry and order.

This simple type of science suits the child until the intermediate period. By such experiences as these, the child gains a familiarity with the more obvious characteristics and habits of natural objects. This forms the foundation for his later scientific work, but it needs to be carried much farther in order to be fully

developed. The type of observation and learning of facts which have been described is suitable to the younger child. The child can begin at least in the kindergarten period, and carry forward this type of work until some time in the intermediate period in school life. At this time he is ready to begin a somewhat more systematic and thorough form of study.

More systematic study is necessary to get at many of the facts of the world about us. Many of the facts of the world about us may be gained by such observations as the child can make in this early scientific work, but the facts must be on the surface and not require too much thought or too sustained observation in order that they may be discovered. For the discovery of many scientific principles, which are commonplace with us, the casual observation of mankind has not been sufficient. Adults miss many of the generalizations which would be very valuable to them and very closely related to their welfare if they were discovered, and the child is less capable of systematic observation and generalization than is the adult. Many illustrations of the weakness of unscientific observation are at hand. The farmer may go on for generations raising corn without discovering the best method of improving his product, although all the facts which are necessary to the discovery of such a method are at hand for his observation, and although it is greatly to his interest to discover them. A little scientific observation and experimentation enables one to discover these princi-

ples. For a long time man suffered disease and death from yellow fever without discovering that it is related to the mosquito, although this fact was at hand all the time for observation. In many cases also mankind has clung to the first generalization, although systematic observation would prove their falsehood. This is the case with such common superstitions as that a rabbit's foot gives good luck, or that Friday and 13 are likely to bring bad luck.

Science is based on systematic observation and experimentation. The means to a more scientific and reliable form of procedure, leading to better established principles, have been incidentally indicated. They consist in a systematic observation of facts together with making a record of the results of the observations in order that they may be compared; and in experimenting in order that various possibilities may be tried out and their results compared with one another. Both of these methods imply that one starts out with a definite problem in mind, and conducts his observation or his experimentation in such a way as to gain an answer to the problem. One must be able to control his investigations and guide them continuously and systematically toward the solution of a particular question. The ability to carry on such scientific work is one which develops gradually, and does not appear until the child has at least reached the period of his intermediate school life.

Systematic observation leads to generalization con-

cerning the sun's movements. The way in which observation and experimentation make generalizations more exact, and lead to the discovery of relationships which would otherwise be unnoticed, may be illustrated by two concrete cases. The first is an illustration of more exact generalization through systematic and directed observation. In certain realms we cannot carry on experiments because the forces which are to be observed or the events which are to be recorded are beyond our control. In astronomy, for example, we have to confine ourselves to the observations of events as they take place. Thus, a child may notice certain general and superficial facts, but may require controlled observation to carry his generalizations beyond the more rough-and-ready ones. One notices without giving especial attention that the sun rises in the east and sets in the west, and that it is higher in the summer than it is in the winter. To find out more exactly the movements of the earth in relation to the sun, one may make a systematic study of the shadows which are cast by some object. Suppose the child marks on the floor the position of the shadow of the corner of the window, or of some object upon the window, from day to day. If he keeps a record of this position, he will notice that it has the same east and west position at a particular hour every day, but that it shifts north and south with the change of the season. This will lead then to a more definite and exact statement of the movement of the sun.

Exact science often requires the use of number. Take another problem, — the relationship of the length of the shadow to the height of the object which casts it. One may readily recognize, without using any exact means of measurement, that the taller object casts the longer shadow, but the comparison of different objects and the shadows which they cast enables one to make the generalization that the shadow is exactly proportional to the height of the object which casts it. This illustration shows how the more exact generalizations in science require the use of numbers, just as we found that the recognition of the longer stretches of time necessitates putting time into terms of numbers. This is one reason why the more exact and comprehensive forms of generalization cannot be made until the child has attained a certain degree of mental development.

In experimentation causes are studied by introducing or varying one at a time. Often the facts which are needed for the solution of a scientific problem can be obtained more quickly by an experiment than by waiting until they present themselves for observation. An experiment also has another advantage, which is that the particular fact which we wish to observe can be made to appear singly; or at least we can observe the effect of certain conditions by varying those conditions while others remain constant. Suppose that we wish to determine what conditions are necessary for the best growth of plants. We may, through experi-

ment, try out one condition after another, taking one at a time. We may vary the amount of water supply, or the amount of air, or the amount of sunlight, or the kind and amount of fertilizer with which the plant is furnished; and by keeping an exact record of the changes which we introduce and of the results which follow in the condition and growth of the plant, we may gain an exact generalization as to the bearing of these different factors.

Science is based on the attempt to solve problems. These illustrations indicate, as has already been remarked, that scientific investigation consists in carrying on systematic observations and experiments to answer certain problems. The importance of this problem-solving process for the education of the child, particularly in natural science, has been very greatly emphasized, and with good reason. If the child merely learns by rote those conclusions which have been reached by others, he may gather a very large amount of information, but he will not be forming those habits of thought and action which will enable him the better to meet those problems which may confront him in the vocational or other situations of his life. The presentation of the work in such a way that it will arouse questions in the mind of the child and lead him to an investigation of these questions is of great importance.

The discovery of problems is also important. The illustrations which have been used and the principles which have been indicated bring out also another form

of scientific work. The ability to do work of a scientific character depends not merely upon the capacity for working out problems, but also, and perhaps in a still more important degree, on the ability to see problems where they have not before existed to the mind of other observers. The successful scientist, or the successful investigator in any field, is the person who sees questions in what the ordinary individual takes as a matter of course. This principle is to be applied both in the natural sciences or in the sciences in a technical sense, and in any field of human endeavor which admits of study by scientific methods. It applies to the problems of everyday life. The conditions which are favorable or unfavorable to health should be studied by each person for himself, so as to learn what conditions are necessary in order that he shall be kept in the best working condition. This, to a large extent, is a matter for individual investigation. Many problems arise also in each person's vocation which should be attacked by applying the scientific method. Therefore, we should encourage the child to look for problems, to give attention to them when he sees them, and to be alive to the questions which may arise in connection with his own life and with the world about him, in addition to training him in the ability to attack problems when they are pointed out.

Exact generalization is induction. When generalization is carried to the point which has just been described, — that is, when it has been made definite and

exact and has resulted from a scientific form of inquiry, — we call it "induction." Induction, then, is the examination of facts in a systematic fashion in order to determine from this investigation what general principles govern these facts.

The formation of generalizations makes concepts more exact. In the foregoing description, scientific procedure has been described as the formation of generalizations. We may equally well describe it as the development of our ideas or our concepts. When the child learns about the habits of life of an animal, besides developing general principles as to the activities of this animal, he is making fuller and more exact his concept or notion of what the animal is. In similar fashion, when the child is taught about the heavenly bodies and learns the general principles which govern the movements of the moon and of the planets, and distinguishes the movements of these bodies from the comparative fixity of the stars, he is making more exact his conception of the meaning of star and planet and moon.

Science also creates concepts. Scientific work, in fact, goes still beyond the process of rendering concepts more precise and exact. In its higher form it means the actual creation of new concepts to explain the facts as they are observed. Much of the science with which we are familiar uses ideas which could never arise from the sensations or perceptions we get from objects. We may think of our idea of a table, or

of a chair, or of a horse, as developing from the perception of a large number of these objects, and from our experience in using them, or in responding toward them. Such an idea as the atom, however, or of gravitation, or of ether vibrations, we would never get by such sensory experiences. These are creations of the imagination, which are brought into being to explain the events which we do observe in the physical world about us.

This abstract form of science is beyond the comprehension of the child below adolescence. Scientific investigation, and the thinking which results in the development of these more abstract forms of concept, represent a higher stage of development, both in the history of science and in the life of the child, than the simpler classifications and generalizations of those events which can readily be observed. They require a type of imagination which goes beneath the physical facts which impress the senses, and which is dependent on a larger amount of mental development than the young child is capable of. We may say in a general way that the science which has as its aim the explanation of physical facts by the creation of such concepts as these is beyond the child below the age of adolescence, and that only the simpler forms are usually grasped even at this age.

Generalizations need to be tested. We have spoken of the development of scientific principles as though it consisted merely in generalization upon facts, and the

form of intellectual procedure which has its culmination in induction. There is another procedure which is continually used by scientific students, and which is important for all kinds of scientific procedure. One develops general principles, as has been said, by systematically examining the facts and attempting to find a principle which will explain them. After this has been done, however, the generalization is not sufficiently certain until the principle which has thus been found is tested.

Testing a proposed explanation is illustrated in the diagnosis of a pupil's poor work. This double procedure of seeking an explanation for observed facts and then testing the explanation may be illustrated from the experience of a teacher. The teacher discovers that a pupil is doing very poor work. He first casts about in his mind to find the explanation for this fact. He may examine the home life of the pupil to see if he has the proper surroundings and the proper encouragement. He may consider the physical conditions of his life, the amount of his food, sleep, rest, and recreation. He may consider his mental capacity and the question whether the work which is given to him is suited to his general ability, or to the special type of ability which he may possess. He may examine the pupil's interests and see whether he is distracted from his work by other and more powerful interests. He may conclude, as a result of this examination, that the pupil is doing poor work because the

work is not suited to his capacity. This, then, is a generalization. He may now test this generalization by changing the work in such a way that it will seem better suited to the pupil. If this results in improvement he may assume that the hypothesis which was laid down was at least in a measure correct.

Another illustration may be found in the discovery of the planet Neptune. We may take as another illustration an example from the larger scientific world. We are all familiar with the law of gravitation. This is a generalization made upon a large number of occurrences, such as the falling of bodies to the earth, the revolution of the moon about the earth and of the planets about the sun. The law of gravitation received a very remarkable confirmation through its application in one particular case. For a long time the movements of the planet Uranus were found not to be in accordance with the mathematical calculations of its course on the basis of the law of gravitation. Various theories were held to account for this, one of them being that the irregularities in movements were caused by the existence of another planet which had not yet been discovered. Two mathematicians worked out theoretically the position and character of a planet which would account for these irregularities in movement. Astronomers then searched in the region of the sky at which such a planet should be located and discovered the existence of the planet Neptune. By this means the correctness of the theory was confirmed by

the fact that the discovery of an object which previously had not been known was predicted and then actually made.

Deduction supplements induction. This mental process, by which we apply a general principle and work out its implication in the facts of the world about us, is called "deduction." These two forms of thinking are both important in scientific generalization. The one leads to the formations of generalizations; and the other to their testing, and to either their confirmation or rejection.

Summary of the child's development in scientific thinking. Scientific thinking in its highest form may be seen, from the previous description, to be the refinement of forms of thinking which the child uses from the beginning. We must not think of the scientific method as something which is an entirely new development at a certain period in a child's life, and which amounts to a radical change or to the adoption of a new form of intellectual activity. From the earliest years the child begins to make generalizations, to form ideas, and to try out his generalizations by comparison with the facts of his observation. As he grows older, he learns to observe in a more systematic fashion, and to experiment, in order that his generalizations may be broader and more firmly founded. He learns to look for problems in the events which he has before taken for granted. As he grows still older, he sees still deeper problems and can appreciate the

search for concepts or ideas which will serve to illuminate or explain them. He is still making generalizations, he is still forming ideas, but they are of a more abstract character. He learns to see the connection of events which previously have had no connection with one another in his mind. He substitutes for the erroneous conclusions, which have been created by popular opinion to explain facts of experience, a more scientific and exact method. His development proceeds through the application of the general forms of thinking which he has used right along, but the application of them becomes more regularly arranged, follows better principles of procedure, and leads to more reliable results.

The practical and the theoretical in science teaching

The child's interest develops from the practical to the theoretical. We have seen that the child's scientific interest grows out of the desire to generalize and to explain the facts of his experience. He is interested first in the more immediate problems, and his attention is attracted to those situations which have a practical meaning and which call for activity on his part. The interest in the generalizations of science is accompanied by still stronger interest in the applications of these generalizations. The boy's interest in the theoretical principles of physics is subordinate to his interest in wireless telegraphy or in X-rays; and the girl's interest in the principles of chemistry is less keen than

her interest in the application of these principles in cooking.

The interest in practical applications is generally recognized. This interest which pupils manifest in the working of general scientific principles in the world about them is generally recognized by teachers of science in the elementary or the high school and by writers of textbooks for elementary or high-school pupils. A comparison of recent texts with those written less than a generation ago makes this clear. A glance at the illustrations of modern texts will show, besides the diagrams which give a graphic representation of the abstract laws, and the cuts of laboratory apparatus, illustrations of engines and telephones; pictures of forests and lakes; of animals or articles of food.

Science teaching may develop general principles from concrete problems or begin with general principles and then proceed to apply them. While the use of a large amount of concrete material is a common feature of the prevailing methods of teaching, there is coming to be a sharp distinction in the way in which this concrete material is used. The customary method in the past has been to develop first the theoretical and abstract scientific principles and afterward make application of them to the practical concerns of everyday life. For example, the general principles of the mechanical effects of heat are first developed and they are then applied to explain the action of the

steam engine through the expansion of water under the influence of heat.

Beginning with general principles means teaching the sciences separately. A consequence of this method is that the pupil is at first introduced to the highly differentiated branches of science. In the history of scientific development the tendency is toward a greater and greater splitting-up of the whole field, due largely to the interests of those who carry on advanced scientific investigation. When we teach pupils by beginning with the abstract, general principles, it is natural to treat the sciences according to the scheme of classification which has been gradually built up in the course of the development of science. The consequence of this mode of treatment is that the pupil is introduced immediately to highly abstract aspects of the physical world.

Illustration of the opposite procedure. The effort is being made in many quarters to begin at the other end, by introducing the pupil first to typical problems as they arise in everyday life, and gradually developing the abstract generalizations from these. To use the example which has already been cited, one would first, in following out this procedure, introduce the pupil directly to the study of engines, and out of this, and other similar cases, arrive at the general mechanical principles which serve to explain their action.

Concrete situations usually involve several sciences. But there are other principles involved in the expla-

nation of the engine than the mechanical principles of physics. Engines derive their energy from some form of combustion. In steam engines the combustion takes place outside the engine proper, while in gas engines it takes place within the engine; but in both cases the energy is released by changes in the composition of the fuel. The explanation of this phase of the action of the engine requires the grasp of chemical principles. Again, if we investigate the sources of fuel energy, we are led to a study of the way in which complex chemical substances are built up in the growth of plants and stored in the wood of trees, or in the earth in coal or oil. In this study we get into the province of botany and even geology.

Two methods of procedure in this case are open. If we begin with such concrete situations as this, two forms of procedure are open. On the one hand, a full explanation may be sought for all the problems which arise, and the search may be pushed into whatever field may need to be explored to reach the object; or, on the other hand, each practical situation may be used to illustrate only the general principle which is most prominently revealed in it, and the situations may be so selected as to illustrate a progressive and coherent group of principles. Even the second method would not involve the rigid separation of the principles of the various sciences, but would necessitate a continuous intermingling and correlation. For example, to understand the flow of sap in a tree it is necessary

to have some grasp of the physical principle of capillarity.

The general principle is clear; the details remain to be worked out. Just how the relation between concrete situations and general scientific principles will be worked out in detail remains to be seen. This is a problem for the special workers in the field. A widely used text in general science, that by Caldwell and Eikenberry, makes the following division into large topics: (1) the Air; (2) Water and its Uses; (3) Work and Energy; (4) the Earth's Crust; (5) Life upon the Earth. Without committing ourselves to this or any particular choice of subjects, as a final list, it is clear that the general procedure which is here represented, of beginning with the objects of the pupil's environment and the problems which they present, and working from these concrete situations to the general principles which serve to explain them, is the procedure which is in accord with the development of the child's scientific thinking as it has been described in this chapter. It is also fairly clear that this involves some breaking-down of the rigid limits between the highly developed special sciences in the early stages of the student's scientific work. After he has attained some conception of the most important modes of the working of the physical world, he is ready to pursue the special laws of some particular class of facts in greater detail and with greater precision, and to follow some of the more abstract theories and speculations.

QUESTIONS AND TOPICS FOR DISCUSSION

CHAPTER I

1. Show that the different school subjects require of the child different kinds of learning. Think out illustrations of your own.
2. Can you give any evidence that learning, if it is properly supervised and directed, may be made more economical than it would otherwise be?
3. Support by any argument you can the view that a knowledge of the learning process is of value to the teacher.
4. Why is the teacher's childhood experience in the school not sufficient to give this knowledge?

CHAPTER II

1. Give all the illustrations you can of motor habits which are trained in the school.
2. Compare the complexity of the writing movement with the walking movement. Why does the child take longer to learn to write than to walk?
3. What conditions determine the slant of writing?
4. If the same conditions operated, what would be the slant in left-hand writing?
5. What important change in the method of writing has been made in order to conform to the demands of hygiene? How have these demands been met subsequently in a different way?
6. Does "selection" of appropriate movements mean conscious, deliberate selection? If not, what is the method?
7. Give illustrations of inhibition in other kinds of behavior, such as thinking.
8. Illustrate the value of rhythm.
9. Discuss the saying, "Practice makes perfect."
10. What changes in writing accompany the automatization of the movement?
11. Discuss the statement, "If the child is trained in the correct movement, the form of the letters will take care of itself."

12. Compare two methods of stimulating the child to a study of the form of his writing.
13. Illustrate good form in other movements than writing.
14. Compare the two methods of grading the child's work.
15. Why should the correct habit be practiced in all writing? Put this rule in the form of a general principle and illustrate from other subjects of study.
16. Compare the movements made by a child of three years and of a child of eight or nine and inquire whether the fundamental and accessory theory explains the difference.
17. Considering the motor development of the child and the need of making writing automatic, when do you think writing drill should be emphasized most?
18. Discuss the aim of bringing all the children of a grade up to a grade standard and giving none drill who are above this standard.

CHAPTER III

1. Is perceptual learning well described as the acquisition of knowledge? Compare it with learning the facts of history or geography.
2. Give other illustrations to show that perception is not a matter of passively gaining impressions.
3. Look up other definitions of apperception. Is there any reason for retaining the term?
4. What is necessary, besides the presentation of an object to the senses, for the success of object teaching?
5. How are recognition and appreciation related to expression, as illustrated by drawing?
6. On what fact does the difficulty in representing perspective rest?
7. Is the nature of the child's early drawing due solely to his inability to master technical difficulties?
8. Gather or observe some drawings of young children and show what is meant by describing them as symbolic.
9. Compare diagramming with the child's early drawing.
10. Does interpretation as a feature of perception mean the same as the interpretation of a poem?
11. If you have opportunity, practice with some illusion until it is overcome or reduced in amount. The Poggendorf illusion, or the illusion of the circles, mentioned in chapter I of Judd's *Genetic Psychology for Teachers*, are good ones to use.
12. What is the chief motive for improvement in perception? What is the place of formal training in sensory discrimination?

13. If it is available, consult Winch's *Children's Perceptions* (Warwick and York, Baltimore) and find illustrations of any principles of this chapter.
14. Why is sense training to be distinguished from perceptual training?
15. Get some facts, if you can, to show the need of diagnosing and correcting sensory defects.

CHAPTER IV

1. Compare formal drill and the expression or the recognition of meaning in both reading and writing.
2. Should formal drill precede, accompany, or follow the recognition of meaning?
3. Is the statement, "The letters need not be learned first," equivalent to saying, "The letters need not be learned"? Why?
4. Is learning to say over the alphabet what is meant by learning the letters, in question 3?
5. Compare the child's progress in learning to recognize a word with the process in learning the figure in chapter III.
6. Mention three stages in the development of the alphabet.
7. Find an illustration of phonetic drill in a first-grade reader and describe it.
8. Why is the incidental method uneconomical when used alone?
9. Compare the type of eye movement in reading with other types of movement you may be able to observe. Is the reading habit found anywhere else?
10. What other activities correspond to the reading of the sentence as a whole?
11. Why do you think oral reading has been so prominent in the school?
12. Is oral reading of equal value in different stages of the child's development? Why?
13. Measure your rate of reading and your ability to reproduce what is read when reading carefully, at ordinary speed, and rapidly. Try the experiment with different kinds of subject-matter. Do your results agree with the conclusions of the chapter?
14. In what chiefly do the upper grades excel the lower ones in reading?
15. How are the mechanics of reading developed?

CHAPTER V

1. Supplement the discussion in the book by suggesting a method of teaching the child to sing from ear.
2. Test the ability of several people, first, in the recognition of absolute pitch, and, second, in the recognition of intervals. Compare the amount of error in the two cases.
3. Similarly, test the ability of several persons to strike a single note or an interval from the printed score, and compare the errors.
4. Is the discrimination between the whole and the half steps on the musical scale a clearly recognized matter in the mind of the person who has not had special musical training?
5. Can you show that, while the recognition of differences in intervals may not be explicit, the untrained person nevertheless uses the conventional scale as the basis of his recognition of musical melody?
6. What analogy is there in another form of learning already studied to the practice of deferring formal instruction in the scale until the child has gained some reading ability?
7. What is the chief difficulty incident to the use of different keys?
8. Is formal instruction in scales and keys desirable? Why?
9. Name all the cases of rhythmical involuntary activities you can. What inference is suggested?
10. What is there in the appreciation of rhythm in music besides the mere understanding of the meaning of musical notation?
11. Is there a sharp distinction between harmony and disharmony?
12. Why is tone quality not one of the chief objects of musical training in the school?
13. If a child cannot distinguish the pitch of middle C and C sharp, what would you advise regarding his musical training? How would you determine his ability in pitch discrimination?

CHAPTER VI

1. May inaccurate spelling be adequate for reading and not for writing? Illustrate.
2. What are the movements which may be associated in spelling?
3. Why must learning to spell English words be in a measure arbitrary?
4. Examine the words in some spelling book, — or better, two spelling books, an older one and a recent one, — and estimate the proportion of the words that are in common use in the writing of the average person.

5. Why should the child, so far as possible, learn to spell by sound, analogy or rules?
6. Why were the inferences from the Philadelphia investigation not conclusive?
7. Compare the essential principles of drill in spelling and in writing.
8. Cite any illustrations you can of children's enjoyment of drill.
9. Why is a variety of modes of presentation best?

CHAPTER VII

1. Show some of the ways in which the child's imagination is stimulated before he enters school. When does imagination develop?
2. Compare memory and imagination.
3. Illustrate further verbal imagination.
4. Investigate differences in imagery among persons of your acquaintance, and report.
5. Is it correct to say that the child is more imaginative than the adult? Amplify your answer.
6. Is biography history?
7. How do you think of events at different past and future dates? How is the time represented in your mind?
8. Make an experiment upon the time sense of another person by asking him to estimate an interval when he does not know, and another when he does know, that you are observing him.
9. Describe your notion of the following time periods: second, minute, hour, day, week, month, year, century.
10. What does simplicity in early history mean?
11. Make a brief outline of beginning history that would conform to the principles laid down in the chapter.
12. Illustrate methods of developing the simpler form of the historical sense.
13. How is tolerance, historical or otherwise, developed?
14. Is the higher form of the historical sense connected with the grasp of historical development?
15. Illustrate helpful and futile use of sources with high-school pupils.

CHAPTER VIII

1. Can you show that the understanding of physical and commercial geography is dependent upon a knowledge of place geography?
2. What is the practical application of the fact that localization of places is the outgrowth of the sense of bodily position?

3. Relate any case of false orientation you have experienced and interpret your experience. Did false orientation have to be readjusted piecemeal?
4. Compare cases in which your sense of direction was good with cases in which it was poor, and explain the difference.
5. How may the gap between experience with a narrow range of objects and the notion of a broad region be bridged?
6. If maps are used abstractly by adults, why need we try to give them concrete meaning to the child?
7. Outline briefly some lessons in home geography to show their general nature.

CHAPTER IX

1. Make a list of other abstract ideas besides number, — if necessary, after the consultation of a textbook on psychology, — and point out their common feature.
2. Mention other illustrations of objects used as counters.
3. If symbols imply the abstractness of an idea, have we met any abstract ideas in connection with the subjects previously discussed? In what way is number more abstract than these?
4. Show the importance of counting in the child's early number development.
5. Use x and y as number symbols for 10 and 11 in order to carry the unit group to 12 instead of 10; turn the numbers 47 and 58 into terms of this notation. Turn the numbers 12 and 23 into the new notation; multiply and turn the answer into ordinary symbols to prove.
6. Illustrate the multiplication of 4 by 5 by using the two types of concrete experience, measurement, and grouping.
7. Illustrate the number 23 by groups of dots.
8. How far do you think it is possible to go in leading the child to an understanding of the number symbols and processes?
9. Mention several forms of number drill.
10. What are the reasons for treating addition and multiplication before subtraction and division? Is the implication that the study of the first two should be completed before the second two are studied?
11. What is a fraction?
12. Why are percentages more easily manipulated than other fractions?
13. What is the essential difference between algebra and arithmetic?
14. State and illustrate some of the processes which can be employed to modify the form of an equation.

15. Make the following equation concrete by substituting for the algebraic symbols numerical terms and show that the equation $(x+y)(x-y) = x^2-y^2$ holds true.
16. Illustrate a method by which the child may be led to recognize through concrete experience what is the sum of the angles of a triangle and of a quadrilateral.
17. How might the relation of the length of the diameter to the circumference of a circle be approximately determined by a concrete method?
18. Compare the stages in the solution of an original problem in geometry with the stages in the reasoning problems given by Dewey in *How We Think*, chapter VI.
19. Apply to some other field the distinction between problem solving and following another's demonstration.
20. Look up some problem in arithmetic, and distinguish between the understanding of the conditions of the problem and of the mathematical process used in solving it.

CHAPTER X

1. Give three scientific generalizations and three generalizations in other fields.
2. Mention other expectations which the child forms and which imply some generalization upon his experience without an explicit formulation.
3. Collect and compare definitions from children of different ages, or find instances in child-study literature.
4. Can you show the effect of names in stimulating concept formation from your experience in this course?
5. Compare your explanation of some fact in a field in which you have studied with the explanation you can give of a fact in an unfamiliar field.
6. State some popular beliefs which rest on superficial observation.
7. Give one other illustration than those given in the book of the light thrown by scientific study upon the affairs of everyday life.
8. How much original investigation of a scientific sort do you think the child can do independently? Is there any intermediate type of investigation between independent research and learning from others?
9. Give an illustration of your own of experimentation.
10. Show that induction and deduction may be used in the investigation of some practical problem.
11. Is it correct to say that the child has no theoretical interest?
12. Do you think the natural order is from the theoretical interest to the practical application or the reverse?

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